Exam II, Math 20C
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February 26, 2010

Name:
Student ID:
Section Number:

Note: There are 3 problems on this exam. You will not receive credit unless you show all your work. No books, calculators, notes or tables are permitted.

I. (40 points) Let $f(x, y)$ be the function given by

$$f(x, y) = x^3 + y^3 - 3xy + 1\,.$$

(1) Determine the critical points of $f(x, y)$.
(2) Classify the critical points of $f(x, y)$ into local maxima, local minima, and saddle points, respectively.
(3) Determine the global maximum and minimum points of $f(x, y)$ on the (closed and bounded) domain

$$D = \{(x, y) \,|\, 0 \le x\,, \quad 0 \le y\,, \quad x + y \le 1\}\ .$$

(4) Use the method of Lagrange multipliers to find the minimum value of $f(x, y)$ along the curve given by the equation $3xy = 1$.

II. (30 points) The temperature at a point (x, y, z) is given by

$$T(x,y,z) = 200e^{-x^2-3y^2-9z^2},$$

where T is measured in Celsius degrees and x, y, z are measured in meters.

(1) Find the rate of change of temperature at the point $P(2,-1,2)$ in the direction toward the point $Q(3,-3,3)$

(2) Find the direction in which the temperature increases the fastest at the point $P(2,-1,2)$.

(3) Find the equation of the plane tangent to the surface consisting of all those points where the temperature is equal to $200e^{-1}$ Celsius degrees at the point $R(1,0,0)$.

III. (30 points) Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function given by

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0)\,. \end{cases}$$

(1) Compute $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ and determine whether the function f is continuous at $(0,0)$ or not.

(2) Compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ and determine whether the function f is differentiable at $(0,0)$ or not.