# MATH 205 (TOPICS IN ALGEBRAIC NUMBER THEORY) - SPRING 2013

**Professor:** Cristian D. Popescu

**Course Topic:** Global Fields

**Course Description:** The main goal of this course is to understand global fields (finite field extensions of $\mathbb{Q}$ or $\mathbb{F}_p(T)$) from an adèlic (algebraic-topological) point of view. The course will be an introduction to adèlic methods in number theory.

The adèlic methods in number theory were introduced by Chevalley (1940) and Artin-Whaples (1945) for the purpose of simplifying and clarifying (the fairly new at the time) global and local class field theory and the relation between the two. The main idea is to associate (in a functorial way) to any global field $F$ a locally compact topological ring $A_F$ (called the ring of adèles of $F$) and express the arithmetic properties of $F$ and those of its finite abelian extensions in terms of certain algebraic-topological properties of $A_F$. The ring $A_F$ is cleverly constructed by patching together the completions of $F$ with respect to all its (equivalence classes of) metrics.

The adèlic methods have played a crucial role in most of the major advances made in number theory since the 1940s. For example, in his PhD thesis (Princeton, 1950), Tate used harmonic analysis on the locally compact group $GL_1(A_F)$ to express certain Hecke $L$–functions associated to $F$ in terms of so-called zeta integrals with respect to a Haar measure on $GL_1(A_F)$ and give a very elegant proof of the meromorphic continuation and functional equation for the $L$–functions in question. The far reaching (and highly conjectural) Langlands Program, which is formulated in adèlic language and whose mighty goal is to understand the absolute Galois group $G_F$ of the global field $F$, can be viewed as a vast non abelian generalization of class field theory and Tate's thesis.

**Background requirements:** Familiarity with local fields (as in Alina Bucur's Math 204 - Winter 2013), algebra (as in Math 200) and basic point set topology. The course will start with a brief review of local fields.