Course announcement – Math 204B (Winter 2018), Math 204C (Spring 2018)

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Math 204B (Winter 2018) – Local fields and local class field theory

This is a course on local fields (locally compact fields). Local fields arise naturally as completions of global fields (number fields or function fields over finite fields, studied in Math 204A) with respect to their various metrics. Their study (from an algebraic and topological point of view) is crucial for our understanding of the arithmetic of global fields, which is one of the main goals of number theory. The course will start with basic local field theory: definition, construction, classification, additive and multiplicative structure, basic properties of finite extensions of local fields. The course will culminate with local class field theory, which is the study of the maximal abelian extension of a general local field. I am planning on presenting two approaches to local class field theory, the cohomological approach (due to Artin-Tate and Nakayama) and the very elegant and explicit formal group approach (due to Lubin-Tate.) The students will be expected to read quite a bit of material on their own, as at some point the lectures will only offer sketches of proofs of the relevant results.

Background requirement: Math 204A, Math 200, curiosity and an open mind.

Recommended texts: Serre's *Local fields;* Iwasawa's *Local Fields;* Cassels and Fröhlich's *Algebraic Number Theory;* Neukirch's *Algebraic Number Theory.*

Math 204C (Spring 2018) – Global class field theory

Global class field theory is the study of the maximal abelian extension of a general global field (number field or function field over a finite field). This course will focus on describing the Galois group of this maximal abelian extension as well as the Galois groups of all the finite intermediate extensions in terms of a topological group canonically associated to the global field in question: its idèle-class group. This strikingly elegant description is probably the most important achievement in number theory of (roughly) the first half of the XX-th Century and it builds upon work of many brilliant mathematicians, including Kronecker, Weber, Hilbert, Takagi, Artin, Hasse, Chevalley, Tate. Unfortunately, unlike the local case (see Math 204B and Lubin-Tate formal groups), the problem of constructing explicitly the maximal abelian extension of a general global field is still wide open: we only known how to do this for ***Q***, for quadratic imaginary fields, and for function fields over finite fields. I am planning on describing these three known explicit constructions, without detailed proofs – they are all geometric in nature, and involve certain algebraic groups with endomorphism by the rings of integers of the given global field. The theories that ensue are the classical **Kronecker-Weber theory** (base field ***Q***, algebraic group ***Gm***), the theory of **complex multiplication** (base field quadratic imaginary, algebraic group a CM ***elliptic curve***), the theory of **rank 1 Drinfeld modules** (base field an arbitrary function field, algebraic group ***Ga***). Finally, I will make the natural links between the global class field theory of a given global field and the local class field theory of all its completions with respect to various metrics.

Background requirement: Math 204A-B, Math 200, curiosity and an open mind.

Recommended texts: Cassels and Fröhlich's *Algebraic Number Theory;* Neukirch's *Algebraic Number Theory.*