Topics for the Algebra Qualifying Exam – Spring 2006

I. Group Theory

1. **Basic properties and examples** (subgroups; homomorphisms; cosets; normal subgroups; quotients etc.)
2. **Free Semigroups. Free Groups. Presentations.** (free semigroups; free groups; universal properties; generators and relations; finitely presented groups.)
3. **The isomorphism theorems. Direct and semidirect products. Extensions** (the isomorphism theorems; direct products; automorphisms of groups; semi-direct products; extensions of groups.)
4. **Group actions on sets** (general definitions and results; permutation groups.)
5. **The Sylow theorems and applications** (Sylow's theorems; applications to the classification of groups of small order.)
6. **Normal series. Solvable and nilpotent groups** (normal series; solvable groups; nilpotent groups.)

II. Ring Theory

1. **Basic properties and examples** (rings, ideals, quotient rings, universality, isomorphism theorems, Noetherian rings etc.)
2. **Arithmetic properties of integral domains** (Euclidean domains, Principal ideal domains, Unique factorization domains.)
3. **Arithmetic in polynomial rings** (irreducibility criteria, Gauss's content lemma, arithmetic properties of polynomial rings etc.)
4. **Rings of fractions in the commutative setting** (definition, basic properties, universality property, contraction and extension of ideals, transfer of arithmetic properties from rings to their rings of fractions, localizations at prime ideals etc…)

III. Module Theory

1. Basic Properties and Examples (modules, submodules, quotient modules, morphisms, isomorphism theorems; direct sums, direct products and their universality properties; exact sequences etc.)
2. Free modules (basic properties and examples, universality property, rings with and without the "independent rank property (IRP)" etc.)
3. Projective and Injective modules (equivalent definitions, examples, "enough projectives", divisible groups, "enough injectives", direct sums/products of projective/injective modules etc.)
4. Modules of Fractions (basic definitions, properties and examples; localization, characterization of exactness in terms of localization, fractions of projective/injective/free modules, fractions of direct sums/products of modules etc.)

5. Tensor products (balanced maps, universality property, tensor products of morphisms, tensor products and direct sums/products, tensor products and fractions/localization, tensor products and Hom etc.)
6. Flat modules (basic definitions and examples, flatness and projectivity/injectivity, flatness and fractions/localization, flatness and torsion etc.)
7. Finitely generated modules over Principal Ideal Domains (Noetherian rings and modules – basic properties and examples, structure theorem for f.g. modules over PIDs and its main consequences, flat/projective/injective modules over PIDs etc …)

Note. For I-III above, your main source of information should be your lecture notes (all the definitions, examples, counterexamples, and theorems discussed in class will be part of the examination material). When studying for the exam, you are strongly advised to combine your lecture notes with the relevant sections in "Abstract Algebra" by Dummit and Foote (Third Edition). Also, you are strongly advised to cover one more time the homework assignments and final exams for Math 200A-B (Fall 2005, Winter 2006), which can be found at the following websites.

http://www.math.ucsd.edu/~cpopescu/Fall05/Math200A/200A.html

http://www.math.ucsd.edu/~cpopescu/Winter06/200B/200B.html

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