

Practice Problems

1. :

(a) Determine whether the following limit exists:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2}$$

(b) Determine whether the function is continuous at $(0, 0)$:

$$f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

2.:

(a)

$$f(x, y) = \frac{x \cos y}{x - y}$$

Compute the partial derivatives $\frac{\partial f}{\partial x}$, and $\frac{\partial f}{\partial y}$.

(b)

$$f(x, y) = x^3 y - 4x y^2.$$

Find $\frac{\partial^2 f}{\partial x \partial y} (1, \frac{1}{2})$.

(c) Find $\frac{\partial f}{\partial t}$ at $t = 0$ where

$$f(x, y) = e^{xy} + y^2, \quad \text{and} \quad \mathbf{r}(t) = \langle \sin t, (t + 1)^2 \rangle .$$

3.:

(a) Find an equation of the tangent plane to the surface

$$\frac{1}{y} - x^2 y + z = 0,$$

at $(-2, 1)$.

(b) Find a linear approximation of the function

$$f(x, y) = \sqrt{4 - x^2 - 2y^2},$$

at $(1, -1)$.