Math 205 – Topics in Number Theory (Winter 2015)

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An Introduction to Iwasawa Theory

Iwasawa theory grew out of Kenkichi Iwasawa's efforts (1950s – 1970s) to transfer to characteristic *0* (number fields) the characteristic *p* (function field) techniques which led A. Weil (1940s) to his interpretation of the zeta function of a smooth projective curve over a finite field in terms of the characteristic polynomial of the action of the geometric Frobenius morphism on its first *l*-adic étale cohomology group (the *l*-adic Tate module of its Jacobian.) This transfer of techniques and results has been successful through the efforts of many mathematicians (e.g. Iwasawa, Coates, Greenberg, Mazur, Wiles, Kolyvagin, Rubin etc.), with some caveats: the zeta function has to be replaced by its *l*-adic interpolations (the so-called *l*-adic zeta functions of the number field in question) and the *l*-adic étale cohomology group has to be replaced by a module over a profinite group ring (the so-called Iwasawa algebra). The link between the *l*-adic zeta functions and the Iwasawa modules is given by the so-called "main conjecture" in Iwasawa theory, in many cases a theorem with strikingly far reaching arithmetic applications.

My goal in this course is to explain the general philosophy of Iwasawa theory, state the "main conjecture" in the particular case of Dirichlet *L*-functions, give a rough outline of its proof and conclude with some concrete arithmetic applications, open problems and new directions of research in this area.

Background requirements: solid knowledge of basic algebraic number theory (the Math 204 series). I will review certain topics in algebraic number theory as needed.

Bibliography: I will not follow a textbook. However, L. Washington's "An introduction to cyclotomic fields" and S. Lang's "Cyclotomic Fields I and II" are good sources of information.