Math 207b Spring 2007: Representations of Quivers
MWF 12-12:50, 5829 AP&M
Professor D. Rogalski

1. Contact Information

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Office hours: Stop by anytime. After 2pm is usually good unless I attend a seminar.

2. Class Description

This course will be an introduction to the representation theory of quivers, starting from the very beginning of the subject. 207b will not be a continuation of the current course 207a, noncommutative projective geometry, as was previously announced. The prerequisite for 207b will be only some background in graduate level algebra, equivalent to past or current enrollment in the Math 200 or math 202 series.

A quiver is just a directed graph, and a representation of it is just an assignment of a finite dimensional vector space V(x) to each vertex x and a linear transformation from V(x) to V(y) for each arrow from x to y. This simple idea pops up in many contexts, and representation theory of quivers has connections and applications to areas as diverse as combinatorics (the properties of Littlewood-Richardson coefficients), the theory of finite dimensional algebras (a representation of a quiver is the same thing as a representation of the corresponding path algebra), algebraic geometry (moduli spaces of representations), and of course representation theory more generally.

Much of the subtlety in the subject comes from the difficult problem of understanding what the indecomposable representations for a given quiver look like. The main goal of the course will be to develop all of the theory needed to prove Gabriel's theorem, which characterizes those quivers which have only finitely many different indecomposables up to isomorphism. In the remaining time we will survey some other interesting results.

3. References

I. Our main reference for the first part of the course are some notes by Harm Derksen from a course on quivers he taught at Michigan. These may be found at

http://www.math.lsa.umich.edu/ hderksen/math711.w01/math711.html

We plan to follow Lectures 1-5 of these notes quite closely, up through the proof of Gabriel's theorem. After that we may cover some other selected parts of Derksen's notes (there are 13 lectures). I recommend you print out lectures 1-5 only to begin with. These first lectures are well-written, exactly at the right level for us, and have lots of good exercises. However, be aware that these lectures do tend to have a lot of typos. If something seems wrong to you, it probably is.

II. After the proof of Gabriel's theorem, there are many choices about what to cover, and I will decide later on. We may cover some of the invariant theory and/or some of the material on moduli spaces of representations in Derksen's notes. In general, I will not say much about combinatorics (it is not my expertise) and we will not cover any of the applications to the properties of Littlewood-Richardson coefficients in Derksen's notes. If this is a big interest of yours you should be able to read those sections on your own once we have developed the basic theory.

III. Another possible topic we may cover is the representation theory of tame quivers. For this, we would follow some notes of Bill Crawley-Boevey, "Lectures on Representations of Quivers", which can be found under the heading "Archived Materials" at his home page:

http://www.amsta.leeds.ac.uk/ pmtwc/

In fact, these notes also contain all of the basic material on quivers including the proof of Gabriel's theorem (but a different proof is given.) However, they are written at a more sophisticated level and so we will not begin with them. They do tend to be more polished than Derksen's notes.

Another interesting set of notes by Crawley-Boevey I recommend if you are intrigued by the connections with algebraic geometry is "DMV lectures on representations of quivers, preprojective algebras and deformations of quotient singularities" found on the same home page. We might try to touch on some topics from these notes.

IV. There are several books on the subject of representations of finite dimensional algebras. These tend to work more generally then we are going to (because we are only going to deal with path algebras), and so they are hard to read. They also tend to work very algebraically and do not seem to deal much with the connection to algebraic geometry, which is the part I like. However, if you want to own a comprehensive book on the subject, I suggest

Elements of the the Representation Theory of Associative Algebras, by I. Assem, D. Simson, A. Skowronski

since the paperback version is relatively cheap and I have heard it is decent. However, I don't expect to follow this book at all, though I might use it as a reference (for this reason I am likely to have the library copy checked out myself, sorry, though you are welcome to borrow it.)