1 Research interests

My primary research interests are problems in optimal location of limited resources on graphs. This field includes the classic operations-research problems in capacitated and uncapacitated facility location, k-median location, dynamically relocatable resources, and search problems such as graph sweeping. These problems cross several disciplines, using traditional tools of combinatorial optimization, game theory, and the structural underpinnings of finite semirings.

2 Current Research

My research at present concerns the dynamic resource relocation problem. This problem deals with a resource which can relocate or reconfigure in response to service requests at locations distributed throughout a system. In the most general form, our resource has a finite set of states S, while requests are drawn from a set R. Relocation of the resource is associated with a cost function $c_r : S \times S \to \mathbb{R}$, while service is associated with a cost $c_s : S \times R \to \mathbb{R}$. If a request sequence $r_1 r_2 \ldots r_n$ and an initial state s_0 is served with the successive choice states of $s_1 s_2 \ldots s_n$, the service plan is said to have cost

$$\sum_{i=1}^{n} c_r(s_{i-1}, s_i) + c_s(s_i, r_i)$$

In traditional framings of this problem, the configuration is presented as a graph G, with $S = R = V(G)$ and $c_r(u, v) = c_s(u, v) = d_G(u, v)$. Two questions are frequently asked relating to this cost-valuation:

- What is the minimum non-negative integer k yielding a function $f : S \times R^k \to S$ such that $s_{i+1} = f(s_i, r_{i+1}, \ldots, r_{i+k})$ will always produce minimum-cost selections of $s_1, \ldots, s_n$?
- Given the condition $s_{i+1} = f(s_i, r_{i-\ell+1}, \ldots, r_{i+k})$, what choice of f minimizes the worst-case ratio between the cost of this selection and a minimum-cost selection of $s_1, \ldots, s_n$, and what is the aforementioned ratio?

The first question has been completely answered over undirected graphs with unweighted edges [6, 7]; the second has been subjected to preliminary consideration [5]. My work places both of these questions into a more generalizable framework in which algebraic tools can be used effectively.

The underlying combinatorial optimization problem allows any given request sequence to be represented as a product of representative matrices in the min-sum algebra. While min-sum and the related max-sum algebra have traditionally been explored primarily with respect to geometrical properties, they have been central in several applications to combinatorial optimization and queueing theory problems, including the traveling salesman problem [1] and industrial system control theory [2, 8, 9].

When request-sequences are represented as matrices, whether a minimizing function f exists for a given request-sequence is determined by the existence of a row in the matrix dominated by all other rows, so the value of k in the first question above is determined the maximal number of terms in a product of representative matrices without a dominated row. This can be investigated via walks in the Cayley graph of the semigroup of min-sum matrices. In particular, the presence of cycles in the Cayley graph is induced by eigenvectors of min-sum matrices, which has been the subject of system-control theory investigations, and can be explicitly determined via a method of Cunninghame-Green [10]. The presence of cycles in the Cayley graph indicates the impossibility of choosing a value of k satisfying our criterion, while the absence of cycles necessarily implies the existence of such a k. This consideration of min-sum eigenvectors as the defining feature in a combinatorial optimization problem is profoundly different than traditional matrix investigations used in combinatorial optimization, which consider matrix rank rather than eigenspace [11]. My research has been devoted to identifying the necessary and sufficient conditions for the existence of such cycles in the Cayley graph, and using these conditions to build a novel approach to dynamic location problems based on traditional results in control theory.

3 Areas of future research

There are several compelling areas of active research connected to the dynamic location problem. Most notable among these is the k-server problem, which has been considered largely in the context of algorithm design for the case where only a single request is known, and in which a server must move to the location of the request [3, 12, 4]; these algorithmic results are not generally known to offer the best competitive ratio, and the competitive ratio for lookahead of more than one request is a largely unexplored problem, even in the case of a single server. Competitive ratios may also be able to be placed into the framework of min-sum algebras; since the minimum competitive ratio has been thus far explored only in terms of specific graphs or algorithms, a new algebraic approach may allow for more general determinations of the ratio's bounds.

Certain other artificialities in the traditional problem descriptions, when discarded, allow for a greater generalization of the underlying problem and a greater applicability to real-world service-relocation problems. For instance, in all formulations of dynamic location problems thus far, remote service has either been prohibited or penalized with a cost equal to the movement cost; that is, in the language of our original problem generalization, the service cost $c_s(u, v)$ has been identical to the movement cost $c_r(u, v)$. In investigations peripheral to my current research, I found that allowing c_s to be a constant multiple of c_r displayed a behavioral threshold between admitting of a simple optimization and not admitting of any limited-knowledge optimization at all. Further exploration into both the particulars of this threshold and into the result of more general selections of c_s would allow greater generalization of the dynamic location problem.

In addition, since control theoretic and dynamic optimization investigations explore many similar properties of min-sum matrices, there is a distinct possibility that these results will expand knowledge not only of dynamic optimizability, but also of industrial systems control.

One of my academic priorities is to develop a research program in which undergraduates can be actively involved: optimization problems on graphs are highly suited for this purpose, as they take on a wide array of forms from highly specific problems in algorithm design to extremely open-ended problems of determining necessary lookaheads for large classes of graphs. Students can thus be guided to problems suitable for their abilities and given promising avenues for exploration and discovery. The fundamental papers in this field use largely elementary methods, and introductory examples for dynamic optimization problems are highly visual. These factors make dynamic graph-location problems, as well as other search problems on graphs, ideally suited for introduction into an undergraduate independent study or research program.

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