# Notation Index

 $\exists$  (there exists) Fn-4  $\forall$  (for all) Fn-4  $\ni$  (such that) Fn-4  $B_n$  (Bell numbers) CL-27  $s \sim t$  (equivalence relation) GT-5 $\binom{n}{k}$  (binomial coefficient) CL-15  $\binom{n}{m_1, m_2, \dots}$  (multinomial coefficient) CL-20 BFE(T) (breadth first vertex sequence) DT-8, GT-29 BFV(T) (breadth first vertex sequence) DT-8, GT-29 C(n,k) (binomial coefficient) CL-15 Cov(X, Y) (covariance) Fn-25 DFV(T) (depth first vertex sequence) DT-8, GT-29  $x|y \ (x \text{ divides } y) \quad \text{GT-24}$ DFE(T) (depth first edge sequence) DT-8, GT-29  $\mu_X$  (expectation or mean) Fn-24 E(X) (expectation) Fn-24  $f \circ g$  (composition) Fn-7  $(n)_k$  (falling factorial) CL-9  $F_n$  (Fibonacci numbers) DT-48 |x| (floor) DT-50 (V, E) (simple graph) GT-2  $(V, E, \phi)$  (graph) GT-2  $\mathbb{N}$  (natural numbers) CL-13 n (first n integers) Fn-1 O() (Big oh notation) GT-38 o() (little oh notation) GT-40  $\mathcal{P}_k(A)$  (k-subsets of A) CL-15, Fn-1  $\mathcal{S}(A)$  (permutations of A) Fn-7 PER(A) (permutations of A) Fn-7

Probability notation  $\mu_X$  (expectation, or mean) Fn-24  $\rho(X, Y)$  (correlation) Fn-25  $\sigma_X$  (standard deviation) Fn-25 E(X) (expectation) Fn-24 Cov(X, Y) (covariance) Fn-25 Var(X) (variance) Fn-25 P(A|B) (conditional probability) DT-27 POSV(T) (postorder sequence of vertices) DT-8 PREV(T) (preorder sequence of vertices) DT-8  $\mathbb{Q}$  (rational numbers) Fn-1  $\mathbb{R}$  (real numbers) CL-28, Fn-1  $\rho(X, Y)$  (correlation) Fn-25 Set notation  $\sim A$  (complement) CL-14, Fn-1  $\in$  and  $\notin$  (in and not in) CL-14 A' (complement) CL-14, Fn-1 A - B (difference) CL-14, Fn-1  $A \cap B$  (intersection) CL-14, Fn-1  $A \cup B$  (union) CL-14, Fn-1  $A \oplus B$  (symmetric difference) Fn-1  $A \setminus B$  (difference) CL-14, Fn-1  $A \subseteq B$  (subset) CL-14  $A \times B$  (Cartesian product) CL-4, Fn-1  $A^c$  (complement) CL-14, Fn-1  $\mathcal{P}_k(A)$  (k-subsets of A) CL-15, Fn-1 |A| (cardinality) CL-3, CL-14  $\sigma_X$  (standard deviation) Fn-25 S(n,k) (Stirling numbers) CL-25  $\Theta()$  (rate of growth) GT-38 Var(X) (variance) Fn-25  $\mathbb{Z}$  (integers) CL-13, Fn-1

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