- 1. (30 pts) (a) Find the strictly decreasing function $f: \underline{4} \to \underline{20}$ of rank 50.
 - (b) Determine the lex order rank of the strictly decreasing function 7, 4, 2, 1. Show your work.
- 2. (15 pts) Find the 6-leaf binary (unlabeled rooted plane) tree whose rank is 20. Show your work.
- 3. (55 pts) Define a *web* recursively to be either(i) The simple graph with

$$V = \underline{3} = \{1, 2, 3\}$$
 and $E = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$

(a "triangle") or

- (ii) A simple graph with V = n for some n > 3 such that vertex n has degree 2 and removing n and the two edges joining it gives a web with n − 1 vertices.
 Do the following.
- (a) Draw the 3 webs that have 4 vertices.
- (b) Prove that a web is not a tree.
- (c) Let w_n be the number of webs with vertex set \underline{n} . Write down a recursion for w_n and explain how you got it.

(d) Prove that, for $n \ge 3$ the number of webs with vertex set \underline{n} is $\frac{(n-1)! (n-2)!}{2^{n-2}}$.

Here are some values of b_n and binomial coefficients.

$$b_1 = 1$$
 $b_2 = 1$ $b_3 = 2$ $b_4 = 5$ $b_5 = 14$ $b_6 = 42$

$$\begin{pmatrix} 6\\4 \end{pmatrix} = 15 \quad \begin{pmatrix} 7\\4 \end{pmatrix} = 35 \quad \begin{pmatrix} 8\\4 \end{pmatrix} = 91 \quad \begin{pmatrix} 5\\3 \end{pmatrix} = 10 \quad \begin{pmatrix} 6\\3 \end{pmatrix} = 20 \quad \begin{pmatrix} 7\\3 \end{pmatrix} = 35 \quad \begin{pmatrix} 8\\3 \end{pmatrix} = 56$$
$$\begin{pmatrix} 4\\2 \end{pmatrix} = 6 \quad \begin{pmatrix} 5\\2 \end{pmatrix} = 10 \quad \begin{pmatrix} 6\\2 \end{pmatrix} = 15 \quad \begin{pmatrix} 7\\2 \end{pmatrix} = 21 \quad \begin{pmatrix} 8\\2 \end{pmatrix} = 28 \quad \begin{pmatrix} 9\\2 \end{pmatrix} = 36 \quad \begin{pmatrix} 10\\2 \end{pmatrix} = 45$$