Name: ______________________ **Section Number:** ______

TA Name: ______________________ **Section Time:** ______

Math 20C.
Final Examination
June 10, 2004

You may use one page of notes and a calculator, but no other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

1. Consider the two planes given by $2x - y + z = 5$ and $x + y - z = 1$.

   (a) Find the angle (in radians) between the two planes. You may leave your answer in the form of an inverse trigonometric function of a number.

   (b) Find a vector equation for the line of intersection of the two planes.

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2. A particle moves through space with velocity function $v(t) = \langle -2t, 1, -2t \rangle$.

   (a) Compute the acceleration function of the particle.

   (b) Assuming the particle is located at the origin when $t = -1$, compute the position function for the particle.

   (c) Compute the speed of the particle when it passes through the point $(0, 2, 0)$.

   (d) Set up (but do not compute) the integral to compute the length of this curve from $t = -1$ to $t = 1$.

3. Consider the function $f(x,y) = 2x^3 - 3x^2 + y^2 - 12x + 10$.

   (a) Find the critical points of f.

   (b) Classify each critical point of f as a local minimum, local maximum or saddle point of f.

4. The base radius r and the height h of a right circular cylinder are measured as 3 centimeters and and 9 centimeters, respectively. There is a possible error of 1 millimeter in each measurement. Use a linear approximation to estimate the maximum possible error in the volume computed from these measurements.

5. UPS ground shipping limits package size by requiring the length of the package plus the girth (distance all the way around the package moving perpendicular to the length) be less than or equal to 108 inches. Find the dimensions of the package with largest volume that can be shipped by UPS ground.

6. Consider the iterated integral $\displaystyle\int_{y=0}^{2}\int_{x=\frac{y}{2}}^{1} y\, e^{x^3}\, dx\, dy$.

(a) Sketch the region integration; clearly label each part of its boundary with the appropriate equation.

(b) Evaluate the integral. Reverse the order of integration, if necessary.

7. The gradient of a function $g(x, y, z)$ at the point $(-3, 4, 5)$ is $\langle -2, 1, 2 \rangle$.

(a) Find the values of the partial derivatives g_x, g_y, and g_z at the point $(3, 4, -5)$.

(b) Find the maximum rate of change of g at the point $(-3, 4, 5)$ and the unit vector in the direction which the maximum rate of change occurs.

(c) Find the rate of change of g at the point $(-3, 4, 5)$ in the direction of the point $(-1, 8, 1)$.

8. Consider the iterated integral

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\; dz\, dy\, dx\,.$$

(a) Convert the integral to an equivalent iterated integral in cylindrical coordinates.

(b) Convert the integral to an equivalent iterated integral in spherical coordinates.

(c) Find the common value of the three iterated integrals by evaluating the one you find simplest.