Final Exam Review Problems

1 The temperature at the point (x, y) is given by a function T satisfying $T_x(5, 1) = 3$ and $T_y(5, 1) = -2$. A bug crawling along the plane is located at the point $(t^3 - t - 1, \sqrt{2t-3})$ at time t. Compute the rate of change of the temperature along the bug's path at the time $t = 2$.

2 Find the distance between the planes $6x + 3y - 2z = 0$ and $6x + 3y - 2z = 8$.

3 Find (and classify) the critical values of the function $f(x, y) = x^3 + xy^2 - 13x - 4y$.

4 If a box of girth 100 inches is sitting on the floor, compute the largest possible *exposed* surface area.

[**Note:** the "girth" of a box is the sum of its length, width, and height.]

5 Let $A = (1, 1, 1)$, $B = (2, 8, 1)$, $C = (5, 4, -4)$.

(a) Find the angle between the lines $\overline{AB}$ and $\overline{AC}$.

(b) Compute the area of triangle $\triangle ABC$.

(c) Find the equation of the plane through A, B, and C.

(d) The line through the point $(-2, 5, p)$ is perpendicular to the plane from (c) and passes through one of the points $\{A, B, C\}$. Compute p, and determine which of the three points the line contains.

6 Early yesterday morning I was hiking on the surface $z = 3x^2 + 5xy + 8y$, but my compass wasn't working. I wandered around randomly, and the only thing I know for sure was that I was heading away from the sun (which was in the East), because it was so bright. At one point I came to a sign telling me I was at the point $(-1, 2, 9)$.

(a) If I want to maintain my exact elevation, in which direction should I walk?

(b) After having an energy drink or three, I felt ready to start climbing. If I want to walk up the hill at a 45° incline, starting at the same point, what direction (or directions) could I head in?

(c) After the caffeine crash, I decided to head downhill. If I'm starting at the same point, what's the steepest slope of descent I could head in, and what unit direction should I take?

7 A cone is constructed with diameter 3 cm and height 2 cm, but the variances on the measurements are both 1 mm. Compute the variance for the surface area of the cone.

[**Note:** the area of the slanted side of a cone is $\pi r s$, where s is the slant height.]

8 Find the mass of a lamina in the plane bounded by the lines $y = 0$ and $y = \sin x$, where $\frac{\pi}{4} < x < \frac{3\pi}{4}$ and the density function is $\rho(x, y) = x + 2$.

9 Evaluate the following integrals:

(a) $\iint_R \frac{e^{xy}}{y}\, dA$, where R is the region bounded by the curves $x = 1$, $y = 1$, and $x^3 y = 1$.

(b) $\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} xz\, dz\, dx\, dy$

(c) $\iiint_R z\, dV$, where R is the region $4 \le x^2 + y^2 + z^2 \le 9$ bounded by the planes $z = 0$ and $z = 7 - 2y$.

(d) $\int_0^1 \int_y^1 e^{-x^2}\, dx\, dy$

10 Find the volume of the region between the cone $z = 3r$ and the sphere $\rho = \sqrt{10}$. (You have to choose which coordinate system to work with.)

11 The location of a particle at time t is $(\sin(2t), e^{-t}, \ln(4t + 1))$.

(a) Find the velocity, acceleration, and speed at time $t = 0$.

(b) Set up, but do not evaluate, an integral for computing the total distance travelled by the particle over the interval $0 \le t \le 5$.

(c) Show that the the speed of the particle is never greater than 3 for $t \ge 0$.