Midterm Review! The harder problems are marked with an asterisk (*).

Review Problems

1 Compute the angles between the following vectors:
  (a) $\langle 1,2\rangle$ and $\langle -8,4\rangle$
  (b) $\langle 5,-3,4\rangle$ and $\langle 0,7,-1\rangle$
  (c) $\langle 1,1,0,-1,0\rangle$ and $\langle 3,2,-1,-7,1\rangle$

2 Find the area of the triangle with corners $(0,2,4)$, $(-1,-1,6)$, and $(1,3,1)$.

3 If $\langle 3,4\rangle$ and $\langle 1,x\rangle$ form a 90° angle, find x.

4 Find the equation for the set of all points that are three times as far from $(0,1,2)$ as they are from $(4,1,-2)$.

5 Compute the sine of the angle between the vectors $\langle 2,1,2\rangle$ and $\langle 0,-1,3\rangle$.

6 A parallelepiped has one vertex at $(0,0,0)$ and adjacent vertices at $(1,2,4)$, $(1,3,9)$, and $(1,-4,16)$. Compute its volume.

7 Find the equation of each of the planes described below:
  (a) the plane parallel to $x+7y-9z=5$ and passing through $(2,2,2)$.
  (b) the plane perpendicular to the line of intersection of the planes $x+4y-z=6$ and $2x-y-3z=1$, and passing through $(4,0,4)$.
  (c) the plane parallel to the lines $\langle 3t+5,t,8-t\rangle$ and $\langle 1-t,2-t,t+4\rangle$ and passing through $(5,5,1)$.
  (d) the plane containing the line $\langle 7t+7,5t-5,2t+2\rangle$ and perpendicular to the line $\langle t+\pi,3t-e,10^{100}-2t\rangle$.

8 Find the velocity, speed, and acceleration of the following vector-valued functions:
  (a) $\vec{x}(t)=\langle t^3+9,7\ln t,e^{-2t}-2\rangle$
  (b) $\vec{x}(t)=\langle \sin 3t,-\cos 3t,4t\rangle$
  (c) $\vec{x}(t)=\langle 3t+6,11-4t,12t+9\rangle$

9 Find the arclength of the following curves:
  (a) $\langle \sin 5t,-12t,-\cos 5t\rangle$, over the interval $0\le t\le 2$
  (b) $\langle 3t^2+8,2t^3-7,5-3t\rangle$, over the interval $-1\le t\le 1$
  (c) $\langle 3e^t-8e^{-t}+13,4e^t+5+6e^{-t},23-10t\rangle$, over the interval $0\le t\le 1$

10 Find the minimum speed of the curve $\langle t^3-15t,3t^2-5,3t-2t^3\rangle$.

*11 Four of the eight corners of a cube are $(-1,-1,-1)$, $(1,2,5)$, $(2,7,-6)$, and $(4,10,0)$. Find the coordinates for the center of the cube.
[**Hint:** Look at the distances between the points. How many different distances can exist between two corners in a cube?]

12 Compute each of the following limits:
  (a) $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}$
  (b) $\lim_{(x,y)\to(0,0)} \frac{x^3+2y^2}{x^2+2y^3}$
  (c) $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^5+y^5}$
  *(d) $\lim_{(x,y)\to(1,1)} \frac{x-2y}{x^3-8y^3}$

*13 The vertices of a triangle in the 3rd dimension are at points A, B, and C. Let $\vec{u}$ be the vector from A to B, $\vec{v}$ the vector from B to C, and $\vec{w}$ the vector from C back to A. If $|\vec{u}|=5$ and $\vec{u}\cdot\vec{v}=23$, compute $\vec{u}\cdot\vec{w}$.
[**Hint:** Use the fact that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$ to rewrite $\vec{u}\cdot\vec{w}$.]