Midterm Review! The harder problems are marked with an asterisk (*).

Review Problems

1 Use a linear approximation to estimate the following quantities:
(a) $(1.99)^3 + (1.02)^3 + (2.02)^3$
(b) $(\arctan(1.04))(\arctan(.98))$

2 Find the distance between the point $(1, 2, 3)$ and the plane $3x - 7y + 4z = 0$.

*3 A cylindrical can has its base sitting in the xy-plane. The bottom circle passes through the origin and the opposite point on the top circle lies in the first octant on the plane $x+3y+5z = 9$. Compute the largest possible volume for the can.

4 Find all values of k such that the function $u(x, y) = x^3 + kxy^2$ satisfies Laplace's equation $u_{xx} + u_{yy} = 0$.

5 Identify the critical values of the following functions:
(a) $f(x, y) = x^3 + 2y^3 + x^2y - 3x^2 - 3y^2$
(b) $f(x, y) = xy + \frac{2}{x} + \frac{4}{y}$
(c) $f(x, y) = e^x(x^2 + y^2)$

6 A particle on the surface $f(x, y) = x^2y^2 - 3xy + x + y$ at $(1, 2)$ heads in the unit direction $\vec{u}$. If the directional derivative is 0, compute all possible vectors $\vec{u}$.

7 Compute the gradient of f at the following points:
(a) $f(x, y) = xy + x^2y^2$ at $(-1, 3)$
(b) $f(x, y, z) = x\sin(yz)$ at $(2, \pi, \frac{1}{3})$
(c) $f(x, y, z) = e^{xy+3z}$ at $(1, 3, -1)$
(d) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(2, -3, 6)$

8 Find the directional derivative of f at the point p in the direction $\vec{u}$:
(a) $f(x, y) = 2x^2 - 4xy + y$, $p = (2, 3)$, $\vec{u} = \langle -4, 3\rangle$
(b) $f(x, y, z) = xy - xz + yz$, $p = (2, 3, 5)$, $\vec{u} = \langle 1, 8, -4\rangle$
(c) $f(x, y, z) = e^{xy} + e^{xz} + yz$, $p = (0, 2, -1)$, $\vec{u} = \langle 2, -1, 2\rangle$

*9 A triangle has its vertices on the circle $x^2 + y^2 = 9$. Find the largest possible area for the triangle.
[**Hint:** The area of a triangle with side-angle-side of a, θ, b is $\frac{1}{2}ab\sin\theta$.]

10 Find the direction of maximum increase of the function $f(x, y) = \sqrt{x^2 + 8xy + y^2}$ at the point $(3, 4)$.

11 The wind speed at the point (x, y) is given by $S(x, y) = (x - y)(x + 2y - 3)$. A bug crawls along the path $x(t) = 1 + \sqrt{t}$, $y(t) = 1 - \sqrt{t} + t$. How fast is the wind speed changing on the bug's path at $t = 4$?

12 Compute each of the following integrals:
(a) $\iint 2xy + 2x + y\, dA$, where R is the rectangle $[1, 2] \times [1, 3]$
(b) $\iint 3y - 4x\, dA$, where R is the region between the curves $y = \frac{x}{3}$ and $x = y^2 - 4$
(c) $\iint \frac{x}{y} - \frac{y}{x}\, dA$, where R is the triangle with vertices $(1, 1)$, $(1, 10)$, and $(10, 1)$
(d) $\iint xe^{y^3}\, dA$, where R is the region $x \le y \le 1$, $x \ge 0$

*13 The ellipsoid $2x^2 + y^2 + 3z^2 = 9$ intersects the plane $3x + 2y + 6z = 1$, creating a closed curve. Find the formula for the tangent line to the curve at the point $(1, 2, -1)$.

*14 Find the radius of the largest circle that can fit inside the closed curve $x^6 + 4y^6 = 1$.

15 Find the maximum and minimum values of the function $f(x, y) = 3x^2 - 2xy + 3y^2 - 2x - 10y$ on the region $x^2 + y^2 \le 9$.

16 Evaluate the double integral $\int_0^4 \int_{\sqrt{x}}^2 \cos\left(\frac{x}{y}\right) dy\, dx$.