

Name: _____ Section Number: _____

TA Name: _____ Section Time: _____

Math 10C.
Final Examination
June 9, 2005

Turn off and put away your cell phone.

You may use a calculator, but no other electronic devices are allowed on this exam.

You may use one page of notes, but no books or other assistance on this exam.

Read each question carefully, answer each question completely, and show all of your work.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

1. (4 points) Find the sum of the geometric series $1 + 2z + (2z)^2 + (2z)^3 + \dots$. You may assume that $|2z| < 1$.

#	Score
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2. (4 points) After conducting extensive research, the American Automobile Association has discovered that the cumulative distribution function for the length of time t in years between tuneups for minivans is

$$P(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 - e^{-\frac{t}{4}} & \text{if } t \geq 0 \end{cases}$$

- (a) What is the probability that a minivan will be driven less than two years between tuneups?

- (b) What is the median length of time that a minivan will be driven between tuneups?

3. (4 points)

(a) Find an equation for the plane tangent to the graph of $f(x, y) = 3 + e^{(2x+3y-1)}$ at the point $(-1, 1, 4)$.

(b) Find a linear approximation for $f(-0.8, 1.1)$

4. (6 points) A spider is on a metal plate whose temperature at (x, y) is $T(x, y) = 120 - x^2 - 2y^2$ degrees Fahrenheit. When it is at the point $(3, 2)$, it is anxious to move in the direction in which the temperature *drops* the most rapidly.
- (a) Find the *unit* vector in the direction in which the temperature drops most rapidly at the instant the spider departs $(3, 2)$.

- (b) If the spider mistakenly moves toward the point $(-1, 5)$, what rate of change in temperature will it experience at the instant it departs $(3, 2)$?

5. (4 points) Let $z = f(x, y)$ where $x = g(t)$ and $y = h(t)$ with

$$g(2) = 3$$

$$g'(2) = -7$$

$$f_x(3, 8) = -5$$

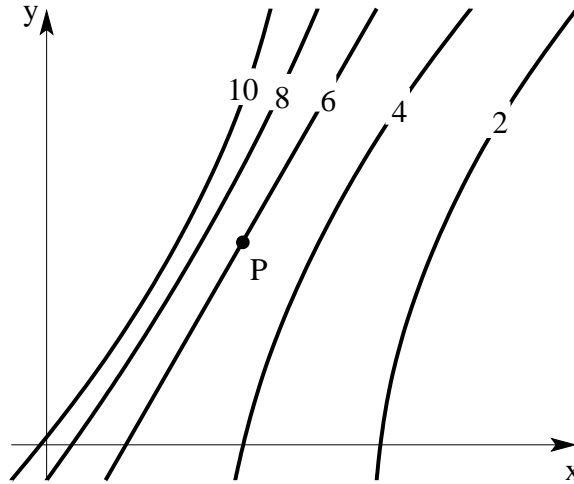
$$h(2) = 8$$

$$h'(2) = 9$$

$$f_y(3, 8) = 2.$$

Find $\frac{dz}{dt}$ when $t = 2$.

6. (6 points) The contour map for a function f of two variables x and y is shown below.



- (a) Determine whether the following partial derivatives are positive, negative, or zero at the point P .

i. f_x

ii. f_y

iii. f_{xx}

iv. f_{yy}

- (b) Sketch the gradient vector $\overrightarrow{\text{grad}} f(P)$ and explain how you chose the direction of this vector.

7. (6 points) Let $g(x, y) = 2x^2 + xy^2 - 2xy + 7$.

(a) Find all the critical points of g .

(b) For each critical point of g , determine whether g has a local maximum, local minimum, or saddle point at that point.

8. (6 points) A company manufactures x units of one item and y units of another. The total cost in dollars, $C(x, y)$, of producing these two items is approximated by the function

$$C(x, y) = 5x^2 + 2xy + 3y^2 + 800.$$

If the production quota for the total number of items ($x + y$) is 39, find the minimum production cost.