Math 20E Homework Assignment 3 ***(updated 5/1/24)***
Due 11:00pm Tuesday, May 7, 2023

1. Evaluate $\displaystyle\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y) = (-x\,y,\ x^2)$ and $\mathbf{c}$ is the path along the unit circle $x^2 + y^2 = 1$ beginning at $(1, 0)$ and ending at $(0, 1)$.

2. Evaluate the line integral $\displaystyle\int_{\mathbf{c}} yz\,dx + xz\,dy + xy\,dz$, where $\mathbf{c}$ consists of the straight-line segments joining $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$.

3. Evaluate the line integral $\displaystyle\int_C (y^2 + 2xz)\,dx + (2xy + z^2)\,dy + (2yz + x^2)\,dz$, where C is an oriented simple curve from $(1, 1, 1)$ to $(0, 2, 3)$.

4. Let $\mathbf{c}(t)$ be a path and $\mathbf{T}(t) = \dfrac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|}$ the unit tangent vector. What is $\displaystyle\int_{\mathbf{c}} \mathbf{T} \cdot d\mathbf{s}$?

5. Let S be the surface determined by the equation $x^3 + 3xy + z^2 = 2$ with $z \geq 0$.
   (a) Find a parametrization $\Phi : D \subseteq \mathbb{R}^2 \to S \subseteq \mathbb{R}^3$.
   (b) Find an equation for the tangent plane to S at the point $(1, 1/3, 0)$.

6. The hyperboloid S with equation $x^2 + y^2 - z^2 = 25$ is parametrized by the mapping

$$\Phi : [0, 2\pi] \times (-\infty, \infty) \longrightarrow \mathbb{R}^3$$
$$\Phi(\theta, u) = 5\,(\cos(\theta)\cosh(u),\ \sin(\theta)\cosh(u),\ \sinh(u))$$

   [See Example 5 in Section 7.3 (pg. 381) of your textbook.]
   (a) Find an equation for the plane tangent to the surface S at $(x_0, y_0, 0)$, where $x_0^2 + y_0^2 = 25$.
   (b) Show that the lines $\boldsymbol{\lambda}_1(t) = (x_0, y_0, 0) + t(-y_0, x_0, 5)$ and $\boldsymbol{\lambda}_2(t) = (x_0, y_0, 0) + t(y_0, -x_0, 5)$ lie in the surface S *and* in the tangent plane to S at $(x_0, y_0, 0)$.

7. Let r and R be positive constants with $0 < r < R$. The mapping

$$\Phi : [0, 2\pi] \times [0, 2\pi] \longrightarrow \mathbb{R}^3$$
$$\Phi(u, v) = ((R + r\cos(u))\cos(v),\ (R + r\cos(u))\sin(v),\ r\sin(u))$$

   parametrizes a torus (or doughnut) T with minor radius r and major radius R.
   (a) Show that all points (x, y, z) in the image T satisfy $\left(\sqrt{x^2 + y^2} - R\right)^2 + z^2 = r^2$.
   (b) Show that the image surface T is regular at all points.

8. Find area of the portion of the unit sphere that is inside the mouth of the cone $z \geq \sqrt{x^2 + y^2}$.

9. The cylinder $x^2 + y^2 = x$ divides the unit sphere S into two regions S_1 and S_2, where S_1 is outside the cylinder and S_2 is inside the cylinder.
   Find the ratio $A(S_1)/A(S_2)$ of the areas of S_1 and S_2.

10. Find the area of the surface S defined by $x + y + z = 1$ with $x^2 + 3y^2 \leq 1$.