240 SYLLABUS

Note: The qual problems will represent a strict subset of this material, and will be similar to what has shown up on previous exams. Please take the time to look over previous UCSD qual problems prior to taking the exam.

- Measure and integration:
  - Basic definitions and properties of σ-algebras (complete measures, σ-finite measures, continuity of measures, etc).
  - Construction of Lebesgue measure on $\mathbb{R}^n$ using cubes. Caratheodory's theorem and the construction of general measures from a premeasure on an algebra. Construction of Borel measures on $\mathbb{R}$.
  - Basic definitions and properties of measureable functions (theorems on pointwise limits, monotone convergence of simple functions, etc).
  - Integration and core convergence theorems (Monotone convergence theorem, Fatou's lemma, Lebesgue's DCT and generalized DCT, Egoroff and Lusin theorems).
  - Modes of convergence such as pointwise, in measure, almost uniform; their relation to each other.
  - Construction of product measures and integration on product spaces. Fubini-Tonelli theorems. Construction of Lebesgue measure on $\mathbb{R}^n$ as a product measure.
- Differentiation of measures and functions:
  - Signed and complex measures. Jordan decomposition.
  - Absolute continuity and the Radon-Nikodym differentiation theorem.
  - Vitali covering Lemma and weak L^1 estimate for the Hardy-Littlewood maximal function. Lebesgue differentiation theorem for $L^1(\mathbb{R}^n)$ and for Radon measures on $\mathbb{R}^n$.
  - Differentiation of monotone functions on $\mathbb{R}$.
  - Bounded variation functions on $\mathbb{R}$ and their Jordan decomposition.
  - Absolutely continuous functions on $\mathbb{R}$ and the fundamental theorem of calculus.
- Point set topology:
  - Basic definitions of PST (open and closed sets, separation and countability axioms, continuity, product topologies, etc).
  - Urysohn's lemma and Tietze extension theorem.
  - Nets and generalized convergence.
  - Compactness and basic consequences (relation to nets, etc). Tychonoff's theorem.
  - The Stone-Weierstrass theorem.
  - The Uryshon metrization theorem.
  - Basic properties of locally compact Hausdorff spaces.
- Basic functional analysis:
  - Definition and basic properties of Banach spaces, Frechet spaces, more general locally convex spaces defined by a family of seminorms. Definition of weak and weak* topologies.
  - Hahn-Banach theorems and basic applications.
  - Baire category theorem and consequences: open mapping theorem, closed graph theorem, principle of uniform boundedness.
  - Weak and weak* convergence. The Banach-Alaoglu theorem.
  - Hilbert spaces. Description of closed subspaces and the Riesz duality theorem.
- L^p spaces:
  - Completeness. Hölder's inequality.
  - Uniform convexity for $1 < p < \infty$. Norm and weak convergence implies strong convergence for uniformly convex spaces.

  - Projection to closed convex sets and variational formula for $1 < p < \infty$.
  - Riesz duality for $1 < p < \infty$. Dual of L^1 when measure space is σ-finite.
  - Young's (convolution) inequality. Minkowski inequality for $L^p(L^q)$.
  - Density of $C_0^\infty(\mathbb{R}^n)$ in $L^p(\mathbb{R}^n)$ for $1 \leqslant p < \infty$. Regularization via convolutions.
  - Complex interpolation theorem.
- Radon measures:
  - Definition of general Radon measures.
  - The Riesz representation theorem.
  - Regularity and approximation properties for Radon measures and associated integrals (Lusin's theorem, approximation by LSC and USC functions, etc).
- Fourier transform:
  - The FT on $\mathbb{R}^n$ and $\mathbb{T}^n$. Completeness of complex exponentials in $L^2(\mathbb{T}^n)$.
  - Basic L^1, L^2, and $\mathcal{S}$ class theory of FT on $\mathbb{R}^n$.
  - Basic formulas for FT (convolution and Parseval identities, inversion formula, Fourier representation of derivatives, Possion summation, etc).
  - Youngs $L^p \to L^{p'}$ mapping property of FT.
  - Heisenberg uncertainty principle for FT.
  - Some basic convergence properties of Fourier integrals (convergence properties of regularized series and integrals, pointwise convergence of of partial sums for regular functions).