FACT SHEET FOR 10C EXAM 1

1. Formulas for Distributions

- If $p(x)$ is a distribution function, then the *cumulative distribution* associated to $p(x)$ is defined to be:

$$P(x) \;=\; \int_{-\infty}^{x} p(y)dy \; .$$

- The *median* of $p(x)$ is the value $x = T$ such that $P(T) = \frac{1}{2}$. In terms of $p(x)$, the median T is given by:

$$\frac{1}{2} \;=\; \int_{-\infty}^{T} p(y)dy \; .$$

- The *mean* of $p(x)$, which we denote by M, is given by the formula:

$$M \;=\; \int_{-\infty}^{\infty} x\, p(x)\; dx \; .$$

2. Geometric Series

- A *finite geometric series* is a sum of the form:

$$S_k \;=\; a + ax + ax^2 + \ldots + ax^{k-1} + ax^k \; .$$

  This series can be explicitly summed, and the answer is:

$$S_k \;=\; a\frac{1 - x^{k+1}}{1 - x} \; .$$

- An *infinite geometric series* is an infinite sum of the form:

$$S_k \;=\; a + ax + ax^2 + ax^3 + \ldots \quad .$$

  As long as $|x| < 1$, this series converges and can be explicitly summed. The answer is:

$$S_k \;=\; a\frac{1}{1 - x} \; .$$

3. Taylor Polynomials

- The *nth degree Taylor polynomial* of a function $f(x)$ at the point $x = a$ is defined by the formula:

$$P_n(x) \;=\; f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{3!}f^{(3)}(a)(x-a)^3 + \ldots + \frac{1}{n!}f^{(n)}(a)(x-a)^n \; .$$

  Here $k! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (k-1) \cdot k$ is the product of all integers from 1 up to k. Also, $f^{(k)}$ refers to the kth derivative of the function f.

- The main point of a Taylor polynomial is that it is the *unique* polynomial which agrees with the derivatives of f up to order n at the point $x = a$. It is supposed to give an accurate representation of the values of the function $f(x)$ when x is "close" to a.

4. Linear Functions

- A general linear function of two variables always looks like:

$$z \;=\; f(x, y) \;=\; a + mx + ny \;.$$

If the point (x_0, y_0, z_0) lies on graph of f (the plane defined by $z = f(x, y)$), then this formula can also be written as:

$$z \;=\; f(x, y) \;=\; z_0 + m(x - x_0) + n(y - y_0) \;. \tag{1}$$

- Recall that to find the formula for a linear function $f(x, y)$, it suffices to do the following:

  i) Find the x-slope m by fixing the variable y and then computing the value $m = \frac{\text{rise}}{\text{run}}$ in z, x.

  ii) Find the y-slope n by fixing the variable x and then computing the value $n = \frac{\text{rise}}{\text{run}}$ in z, y.

  iii) Now that you have m and n, plug a fixed point (x_0, y_0, z_0) on the graph of $f(x, y)$ into the formula (1) to get the final equation.