Math 174 Final

December 13, 2013

- Please put your name, ID number, and sign and date.
- There are 8 problems worth a total of 200 points.
- **You must show your work to receive credit**.

Print Name: ______________________________

Student ID: ______________________________

Signature and Date: ______________________________

Problem	Score
1	/25
2	/25
3	/25
4	/25
5	/25

Problem	Score
6	/25
7	/25
8	/25
Total	/200

1. (25 pts) Given the following header for a **Matlab** function:

   function [x] = SolvePLU(P,L,U,b,n)

   where P, L, U make up the PLU factorization for an $n \times n$ matrix A ($PA = LU$) and b is a vector, **complete** the function so that it uses the **PLU factorization** to solve the linear system $Ax = b$ for x. Do **not** use Matlab's in-built inverse or matrix-matrix or matrix-vector multiplications.

2. (25 pts) **Circle** the best answer in each part. You do **not** have to show your work in this problem.

   (a) The number of **flops** needed to perform LU factorization on an $n \times n$ matrix is:

   (i) $\mathcal{O}(n)$ (ii) $\mathcal{O}(n^2)$ (iii) $\mathcal{O}(n^3)$ (iv) $\mathcal{O}(n^4)$

   (b) Suppose runtime is mainly influenced by flops. Let A be an $n \times n$ matrix and B a $2n \times 2n$ matrix, both **tridiagonal** and with nonzero diagonal elements. Then each iteration of Gauss-Seidel when solving $By = c$ is this many **times slower** than when solving $Ax = b$:

   (i) 2 (ii) 3 (iii) 4 (iv) 8

   (c) Suppose f is a smooth function and x_0 is a location. Let $h_1, h_2 > 0$ be two small stepsizes with $h_1 = 2h_2$. The absolute error of second order central differencing approximating $f'(x_0)$ when using h_2 is this many **times smaller** than when using h_1:

   (i) 2 (ii) 3 (iii) 4 (iv) 8

   (d) Let $p(x)$ be a polynomial of degree 3 and let x_0, x_1, x_2, x_3, x_4 be **five** distinct nodes. Then the interpolating polynomial of least degree for the data points $(x_0, p(x_0)), (x_1, p(x_1)), (x_2, p(x_2)), (x_3, p(x_3)), (x_4, p(x_4))$ has this **degree**:

   (i) 3 (ii) 4 (iii) 5 (iv) none of the above

   (e) Let f be a smooth function with **two** roots and x_0 an initial guess. If Newton's method converges, it will always converge to the root that is **closest** to x_0.

   TRUE or FALSE

   (f) Gauss-Seidel and Jacobi methods' sequences of approximations are exactly the **same** when using the same initial guess and applied to the same linear system $Ax = b$, where A is **upper triangular** with nonzero diagonal elements.

   TRUE or FALSE

   (g) Trapezoid rule on a **concave down** function gives an **underestimate** of the real integral.

   TRUE or FALSE

   (h) Midpoint rule on a **concave down** function gives an **underestimate** of the real integral.

   TRUE or FALSE

3. (25 pts) For each part below, **find examples** of the specified quantities. Be sure to **justify** that your choices satisfy the listed requirements.

   (a) Let $f(x) = x^2 - 4$. Come up with a **starting interval** $[a_0, b_0]$ such that $[a_2, b_2]$ generated by bisection method satisfies $a_2 \neq a_0, b_2 \neq b_0$.

   (b) Come up with a **sequence** that converges to 5 with order of convergence 3.

(c) Come up with **two** different polynomials of any degree that interpolate the following data:

x	-1	0	1
y	1	2	-1

(d) Come up with **three** data points with distinct nodes such that the interpolating polynomial of least degree has degree < 2.

4. (25 pts) Consider the iterative method whose sequence of approximations satisfies:

$$(2D - L)x^{(k+1)} = b + (U + D)x^{(k)},$$

for solving a linear system $Ax = b$, with $A = D - L - U$, where D is diagonal, L is strictly lower triangular, and U is strictly upper triangular. When

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix},$$

write out the **iteration matrix** and use it to determine whether this iterative method's sequence of approximations will **converge** to the solution of $Ax = b$ for **any** initial guess.

5. (25 pts) Find a, b, c, d, f so that the following is a **free** or **natural** ($S''(\text{endpoints}) = 0$) **cubic spline**

$$S(x) = \begin{cases} 1 - 2x - 3x^2 + ax^3, & \text{if } -1 \leq x < 2, \\ b + c(x-2) + d(x-2)^2 + f(x-2)^3, & \text{if } 2 \leq x \leq 3 \end{cases}$$

6. (25 pts) Consider the table of data:

x	0.9	1	1.1	1.4
$f(x)$	0.54	0.5	0.46	0.4

First write out **Newton's form** for the interpolating polynomial $p(x)$, then use it to **approximate** $f'(1)$.

7. (25 pts) Use **Taylor series** to find the p satisfying

$$\left| \int_0^h f(x)\ dx - \frac{h}{2}[f(0) + f(h)] \right| = \mathcal{O}(h^p).$$

8. (25 pts) Consider

$$y' = 3(t+1)y$$

with $y(0) = 2$. Use **one** step of **predictor-corrector** to compute $y_1 \approx y(0.1)$ with **Midpoint method**

$$y_{i+1} = y_i + hf\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}f(t_i, y_i)\right)$$

as predictor and **Trapezoid method**

$$y_{i+1} = y_i + \frac{h}{2}(f(t_i, y_i) + f(t_{i+1}, y_{i+1}))$$

as corrector.