

Math 274 Final

December 7, 2016

- Please put your name, ID number, and sign and date.
- There are 2 problems worth a total of 50 points.
- **You must show your work to receive credit.**

Print Name: _____

Student ID: _____

Signature and Date: _____

Problem	Score
1	/25
2	/25
Total	/50

1. (25 pts) Let a function g in $[a, b]$ be called strictly convex if, for any subinterval $[c, d] \subseteq [a, b]$, the line L passing through the two points $(c, g(c)), (d, g(d))$ satisfies $L(x) > g(x)$ for all $x \in (c, d)$. Suppose $f(x)$ is a strictly convex function that is continuous in $[a, b]$, $a < b$, such that $f(a) < 0$ and $f(b) > 0$. Let $[a_i, b_i]$ denote the interval constructed by the method of false position at the i th step, for $i = 0, 1, \dots$. If the initial interval $[a_0, b_0] = [a, b]$, prove $b_i = b$ for all i .

2. (25 pts) Let $Ax = b$, where A is a nonsingular $n \times n$ matrix and b a nonzero $n \times 1$ vector. Also let $(A + \delta A)(x + \delta x) = b + \delta b$, where $A + \delta A$ is nonsingular. Suppose $\|\cdot\|$ is a natural norm and $\|\delta A\| < 1/\|A^{-1}\|$. Prove

$$\frac{\|\delta x\|}{\|x\|} \leq \frac{\kappa(A)}{1 - \kappa(A)(\|\delta A\|/\|A\|)} \left(\frac{\|\delta b\|}{\|b\|} + \frac{\|\delta A\|}{\|A\|} \right).$$