Homework #3

1. (a) Starting with the initial guesses of $p_0 = 2$ and $p_1 = 1$, draw how two additional approximations of the root of $f(x) = x^2 - 2$ are calculated under the secant method.

   (b) Find the values of p_2, p_3, p_4 when $p_0 = 2$ and $p_1 = 1$. Also write down the absolute error of the approximation p_4.

2. (a) Starting with the initial guesses of $p_0 = 1$ and $p_1 = 2$, draw how two additional approximations of the root of $f(x) = x^2 - 2$ are calculated under the method of false position.

   (b) Find the values of p_2, p_3, p_4 when $p_0 = 2$ and $p_1 = 1$. Also write down the absolute error of the approximation p_4.

3. Consider $f(x) = x(1 - e^x)$. Note: you will need to store enough decimal places in your computations to get meaningful results.

   (a) Find the multiplicity of the root at $x = 0$.

   (b) Using Newton's method with $p_0 = 0.1$, compute $|p_{n+1}|/|p_n|$ and $|p_{n+1}|/|p_n|^2$ for $n = 0, 1, 2, 3$. Does Newton's method look like it is linearly or quadratically convergent in this case?

4. Suppose p_0 is chosen with absolute error 10^{-1} and we want to achieve absolute error $\leq 10^{-7}$.

   (a) Consider a sequence of approximations with $\frac{|p_{n+1}-p|}{|p_n-p|^{1.62}} = 1$. Find the first n with the desired absolute error. If this sequence of approximations requires 1 function evaluation for each of p_k, how many are needed to calculate $p_0, \ldots, p_n$, for your computed n?

   (b) Consider a sequence of approximations with $\frac{|p_{n+1}-p|}{|p_n-p|^2} = 1$. Find the first n with the desired absolute error. If this sequence of approximations requires 2 function evaluations for each of p_k, how many are needed to calculate $p_0, \ldots, p_n$, for your computed n?

   (c) Which of the two has fewer function evaluations?

5. (a) Construct a sequence that has order of convergence 1 and asymptotic error constant 1/4. Justify your results.

   (b) Construct a sequence that converges to -5 that has order of convergence 2. Justify your results.

   (c) Construct a sequence that has order of convergence 3. Justify your results.

6. Answer True or False for the following questions. If you are not sure of your answer, write a short explanation of your choice.

(a) Newton's method's sequence of generated approximations do not change when $f(x)$ is replaced by a constant times $f(x)$.

(b) The method of false position's sequence of generated approximations do not change when $f(x)$ is replaced by a constant times $f(x)$.

(c) Secant method's approximations $x_2, x_3, \ldots$ do not change when $x_0 = a, x_1 = b$ is replaced by $x_0 = b, x_1 = a$.

(d) The secant method obtains the exact root for its approximation after the first iteration, p_2, using any two distinct initial guesses when $f(x) = ax + b$ with $a \neq 0$.

7. (a) Suppose $f(1) = 4$, $f(2) = 3$, and $f(3) = 1$. Verify that the quadratic polynomial

$$p(x) = f(1)\frac{(x-2)(x-3)}{(1-2)(1-3)} + f(2)\frac{(x-1)(x-3)}{(2-1)(2-3)} + f(3)\frac{(x-1)(x-2)}{(3-1)(3-2)}$$

passes through the points $(1,4), (2,3), (3,1)$. Then simplify it to the form $p(x) = ax^2 + bx + c$.

(b) Use the quadratic formula to find the roots of $p(x)$.

(c) Using the above calculations, find p_3 of Muller's method on $f(x)$ when $p_0 = 1$, $p_1 = 2$, $p_2 = 3$.

8. (Matlab) Using the "cos" command in Matlab, write a Matlab function that inputs a number x and outputs the value $\cos x - x$.

(a) Write a Matlab function that inputs two initial guesses p_0, p_1 and the number of iterations N, and outputs the method of false position's p_N approximation of the root of the function above. Print out or write out the function and turn it in.

(b) Run your program for the starting interval $[0, \pi/2]$ and write down the values of p_2, p_5, p_{10}.

9. (Math 274) Let a function g in $[a,b]$ be called strictly convex if, for any subinterval $[c,d] \subseteq [a,b]$, the line L passing through $(c, g(c)), (d, g(d))$ satisfies $L(x) > g(x)$ for $x \in (c,d)$. Now let f be a strictly convex and continuous function in $[a,b]$, $a < b$, such that $f(a) < 0, f(b) > 0$.

(a) Prove f has a unique root in $[a,b]$.

(b) Prove the approximation after the first iteration, p_2, of the method of false position using $p_0 = a, p_1 = b$ satisfies $p_2 < p$, where p is the root of f in $[a,b]$.

(c) Conclude that all further approximations p_n satisfy $p_n < p$ for $n \geq 2$ and that the right endpoint of the interval used in the method of false position never changes.