Homework #3

1. (b) Using secant method formula,

$$p_2 = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)} = 1 - \frac{(-1)(-1)}{-1-2} = \frac{4}{3} \approx 1.33333333333333$$

and

$$p_3 = p_2 - \frac{f(p_2)(p_2 - p_1)}{f(p_2) - f(p_1)} = \frac{4}{3} - \frac{(-2/9)(1/3)}{-2/9+1} = \frac{10}{7} \approx 1.42857142857143$$

and

$$p_4 = p_3 - \frac{f(p_3)(p_3 - p_2)}{f(p_3) - f(p_2)} = \frac{10}{7} - \frac{(2/49)(2/21)}{2/49+2/9} = \frac{41}{29} \approx 1.41379310344828$$

3. (b) First, Newton's method gives us:

$$\begin{aligned} p_1 &\approx 0.05123933030278592 \\ p_2 &\approx 0.02594642975690419 \\ p_3 &\approx 0.01305718390501691 \\ p_4 &\approx 0.00654987994478523. \end{aligned}$$

We compute the ratios $|p_{n+1}|/|p_n|$:

$$\begin{aligned} |p_1|/|p_0| &\approx 0.512393303027859 \\ |p_2|/|p_1| &\approx 0.506377222410603 \\ |p_3|/|p_2| &\approx 0.503236245886295 \\ |p_4|/|p_3| &\approx 0.501630366274354, \end{aligned}$$

which look like they are converging to 0.5. Thus Newton's method looks to be linearly convergent to 0, with asymptotic error constant 0.5. On the other hand, we compute the ratios $|p_{n+1}|/|p_n|^2$:

$$\begin{aligned} |p_1|/|p_0|^2 &\approx 5.12393303027859 \\ |p_2|/|p_1|^2 &\approx 9.88258861734325 \\ |p_3|/|p_2|^2 &\approx 19.3952019835171 \\ |p_4|/|p_3|^2 &\approx 38.4179597931231, \end{aligned}$$

which do not look to be converging, or perhaps are converging to ∞. Thus Newton's method does not look to be quadratically convergent to 0.

7. (a) Note,

$$\begin{aligned} p(1) &= f(1)\frac{(1-2)(1-3)}{(1-2)(1-3)} + 0 + 0 = f(1) = 4 \\ p(2) &= 0 + f(2)\frac{(2-1)(2-3)}{(2-1)(2-3)} + 0 = f(2) = 3 \\ p(3) &= 0 + 0 + f(3)\frac{(3-1)(3-2)}{(3-1)(3-2)} = f(3) = 1, \end{aligned}$$

so p interpolates the data points $(1,4),(2,3),(3,1)$. Simplified,

$$p(x) = 2(x^2 - 5x + 6) - 3(x^2 - 4x + 3) + \frac{1}{2}(x^2 - 3x + 2) = -\frac{1}{2}x^2 + \frac{1}{2}x + 4.$$

(b) Thus, p has two roots of

$$x = \frac{-(1/2) \pm \sqrt{1/4 + 8}}{-1} = \frac{1}{2} \mp \frac{\sqrt{33}}{2}.$$

(c) Muller's method will take, as p_3, the root closer to $p_2 = 3$. Note

$$\left|\frac{1}{2} - \frac{\sqrt{33}}{2} - 3\right| \approx 5.37228132326901$$

and

$$\left|\frac{1}{2} + \frac{\sqrt{33}}{2} - 3\right| \approx 0.372281323269014,$$

so $p_3 = \frac{1}{2} + \frac{\sqrt{33}}{2}$.

8. (Matlab)

(a) See "hw3afn.m".

(b) We get $p_2 = 0.611015470351657$ and $p_5 = 0.738877768847912$ and $p_{10} = 0.739085129248206$.

9. (Math 274)

(a) Suppose $p < q \in [a,b]$ are two roots of f. Note

$$L(x) = \frac{-f(a)}{q-a}(x-q)$$

is the line passing through $(a, f(a)), (q, 0)$, and $L(x) < 0$ for $x \in [a, q)$. Now f strictly convex implies $L(x) > f(x)$ in (a, q). Note $p \neq a$ since $f(a) < 0$, and $p \in (a, q)$ implies $L(p) > f(p) = 0$, which contradicts $L(p) < 0$. Thus, there cannot be two different roots of f in $[a, b]$.

Now f continuous means, by the Intermediate Value Theorem, f has a root in (a, b). So f has a unique root in $[a, b]$.

(b) Let L be the line passing through $(p_0, f(p_0)), (p_1, f(p_1))$. Then $p_2 \in (a,b)$ is the root of L. But $L(x) > f(x)$ for $x \in (a,b)$, so $f(p_2) < 0$. The Intermediate Value Theorem says a root of f has to be in (p_2, b). Since f has a unique root in (a,b), this root is p, and $p \in (p_2, b)$. This implies $p > p_2$.