Homework #4

1. For each part, find the interpolating polynomial for the data points

$$(-1,2),(1,3),(2,-2).$$

by writing down a linear system involving $p(-1)=2, p(1)=3, p(2)=-2$ and solving it for the unknown coefficients a, b, c:

   (a) (Not due) $p(x) = ax^2 + bx + c$.

   (b) Lagrange form: $p(x) = a\frac{(x-1)(x-2)}{6} + b\frac{(x+1)(x-2)}{-2} + c\frac{(x+1)(x-1)}{3}$.

   (c) Newton form: $p(x) = a + b(x+1) + c(x+1)(x-1)$.

   (d) (Not due) Simplify each and show they are all the same polynomial.

2. Find three different polynomials of any degree interpolating the data points

$$(1,4),(2,-2),(3,1).$$

3. Consider data points $(x_0-h, f(x_0-h), (x_0, f(x_0)), (x_0+h, f(x_0+h))$.

   (a) Find the interpolating polynomial $p(x)$ passing through these points in Lagrange form.

   (b) Show $p'(x_0) = \frac{f(x_0+h)-f(x_0-h)}{2h}$.

   (c) Show $p''(x_0) = \frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}$.

   (d) (Not due) Show $\int_{x_0-h}^{x_0+h} p(x)\ dx = \frac{h}{3}(f(x_0-h)+4f(x_0)+f(x_0+h))$.

4. (a) Write down the divided difference table for the data:

x	-1	0	1
$f(x)$	-2	0	3

   (b) Use your divided difference table to write down the Newton forms for the interpolating polynomials for the data points:

      i. (Not due) $(-1,-2),(0,0)$.

      ii. (Not due) $(0,0),(1,3)$.

      iii. $(-1,-2),(0,0),(1,3)$.

   (c) Modify your divided difference table to add in the data point $(3,3)$ and use it to write down the Newton form for the new interpolating polynomial.

5. (Not due) Let p be the piecewise linear interpolating polynomial for data with nodes $x_j = 2j/n$, $j = 0, \ldots, n$ and values from the underlying function $f(x) = \sin \pi x$.

   (a) For $n = 100$, find the value of $p(1.431)$.

   (b) Find n such that the absolute error in $[0,2]$ is guaranteed to be less than 10^{-5}.

6. (Matlab)

   (a) Write a Matlab function that inputs x, a vector of x-coordinates of data points (nodes); y, an array of y-coordinates of data points; n, the total number of data points; and z, one location on the x-axis. Have it output the value of the interpolating polynomial for the data points, computed using Lagrange form, at z. Write out or print out your function and turn it in.

   (b) Use your program to approximate $f(2)$ for the table of data

x	0	1	4	9	16
$f(x)$	0	1	2	3	4

   Write out or print out your results and turn them in.

7. (Matlab) (Not due)

   (a) Write a Matlab function that inputs x, a vector of x-coordinates of data points (nodes); y, an array of y-coordinates of data points; n, the total number of data points; and z, one location on the x-axis. Have it output the value of the interpolating polynomial for the data points, computed using Newton form, at z. Write out or print out your function and turn it in.

   (b) Use your program to approximate $f(2)$ for the table of data

x	0	1	4	9	16
$f(x)$	0	1	2	3	4

   Write out or print out your results and turn them in.

8. (Math 274) Prove uniqueness of polynomials of degree ≤ 3 satisfying

x	x_0	x_1
$f(x)$	y_0	y_1
$f'(x)$	z_0	z_1

   when $x_0 \neq x_1$, by studying the derivative of $p(x) - q(x)$, for two such polynomials p, q, and counting the number of roots from the data and from Rolle's Theroem, and applying Fundamental Theorem of Algebra.