Homework #3

1. (b) Plugging in data points, we get the system of linear equations:

$$\begin{aligned} a &= 2 \\ b &= 3 \\ c &= -2. \end{aligned}$$

So

$$p(x) = \frac{1}{3}(x-1)(x-2) - \frac{3}{2}(x+1)(x-2) - \frac{2}{3}(x+1)(x-1).$$

(c) Plugging in data points, we get the system of linear equations:

$$\begin{aligned} a &= 2 \\ a + 2b &= 3 \\ a + 3b + 3c &= -2. \end{aligned}$$

Solving from top to bottom, $a = 2$ and $b = (3-2)/2 = 1/2$ and $c = (-2-2-3/2)/3 = -11/6$. So

$$p(x) = 2 + \frac{1}{2}(x+1) - \frac{11}{6}(x+1)(x-1).$$

3. (a) The Lagrange form of the Lagrange polynomial takes the form:

$$\begin{aligned} p(x) &= f(x_0-h)\frac{(x-x_0)(x-x_0-h)}{(-h)(-2h)} + f(x_0)\frac{(x-x_0+h)(x-x_0-h)}{(h)(-h)} + \\ &\quad f(x_0+h)\frac{(x-x_0+h)(x-x_0)}{(2h)(h)} \\ &= f(x_0-h)\frac{(x-x_0)(x-x_0-h)}{2h^2} - f(x_0)\frac{(x-x_0+h)(x-x_0-h)}{h^2} + \\ &\quad f(x_0+h)\frac{(x-x_0+h)(x-x_0)}{2h^2}. \end{aligned}$$

(b) Taking a derivative, plugging in x_0, and simplifying:

$$\begin{aligned} p'(x_0) &= f(x_0-h)\frac{-h}{2h^2} - f(x_0)\frac{-h+h}{h^2} + f(x_0+h)\frac{h}{2h^2} \\ &= -\frac{f(x_0-h)}{2h} + \frac{f(x_0+h)}{2h} \\ &= \frac{f(x_0+h) - f(x_0-h)}{2h}. \end{aligned}$$

4. (a) The divided difference table looks like:

-1	-2		
		2	
0	0		1/2
		3	
1	3		

   (b) iii. The Newton form for the Lagrange polynomial is:

$$p(x) = -2 + 2(x+1) + \frac{1}{2}(x+1)x.$$

6. (Matlab)

   (a) See "hw4afn.m".

   (b) The function gives an approximation of 1.57222222222222 for $f(2)$.

9. (Math 274) Given p, q two polynomials of degree ≤ 3 interpolating the data, let $r = p - q$, a polynomial of degree ≤ 3. Then $r' = p' - q'$, a polynomial of degree ≤ 2. From the data, $r'(x_i) = p'(x_i) - q'(x_i) = z_i - z_i = 0$, for $i = 0, 1$, so, since $x_0 \neq x_1$, r' has at least 2 roots. In addition, $r(x_i) = p(x_i) - q(x_i) = y_i - y_i = 0$, for $i = 0, 1$, so r has roots at x_0, x_1. Thus, by Rolle's Theorem, r' has a root at some ξ between x_0, x_1. So r' has at least three roots: x_0, x_1, ξ. But this is more than its degree, so Fundamental Theorem of Algebra implies $r' \equiv 0$. Integrating, r is a constant function, and, since it has roots at the nodes, $r \equiv 0$ and $p \equiv q$.