Homework #6

1. (a) Consider the data points

$$(-1,-1.1),(-1,-1),(0,0.9),(0.5,1.8),(1,3.2).$$

Write down the normal equations for linear least squares.

(b) Solve the normal equations to get the best fitting line in the least squares sense.

(c) Write down the normal equations for quadratic least squares. You do not need to solve this linear system.

2. Consider the n data points

$$(0,y_1),(0,y_2),\ldots,(0,y_n).$$

Find the constant function best fitting this data in the least squares sense. What is another name for this constant?

3. Determine the best quadratic least squares approximation for the data

$$(-1,2.04),(-0.5,1.23),(0,1.01),(0.5,1.28),(1,1.99).$$

4. Derive from minimization the normal equations, in matrix form, for best least squares aproximation of data points $(x_1,y_1),\ldots,(x_n,y_n)$ by a quadratic $a_0+a_1(1+x)+a_2(1+x+x^2)$ under the inner product $<\vec{v},\vec{w}>=\sum_{i=1}^{n} c_i v_i w_i$, where c_i, $i=1,\ldots,n$ are given positive numbers.

5. (a) Consider the data points

$$(0,1),(0.1,1.01),(0.2,1.04),(0.3,1.09),(0.4,1.16).$$

Approximate the first derivatives of f at each node location using the best of either forward, backward, or central differencing.

(b) Find the exact absolute errors at each node location using the fact that the data comes from $f(x)=1+x^2$.

(c) Use the error expression for central differencing to explain why its approximations were exact.

(d) Approximate the second derivative of f at $x=0.2$ using central differencing.

6. (a) Let $f(x)=\sin x$. Approximate $f'(1)$ using central differencing with $h=0.1,0.05,0.025$.

(b) Calculate the exact absolute errors $E(h)$ for each of your approximations.

(c) Calculate $E(h)/E(h/2)$ for each of $h=0.1,0.05$. What integer do you think this converges to as $h\to 0$?

(d) Let $E(x,h)$ be the absolute error of the central differencing approximation of $f'(x)$ using stepsize h. Estimate h such that $E(x,h) \leq 10^{-10}$ is satisfied for all x.

7. Use Taylor series to show $\frac{f(x+h)-2f(x)+f(x-h)}{h^2}$ as an approximation of $f''(x)$ has absolute error $\mathcal{O}(h^2)$.

8. (Matlab)

(a) For given $f(x)$, write a Matlab function that inputs x, and outputs the value of $f(x)$. Then write a Matlab function that inputs x and stepsize h, and outputs the central differencing approximation of $f''(x)$:

$$\frac{f(x+h)-2f(x)+f(x-h)}{h^2},$$

calling your previous function when a value of f is needed. Print out or write out this latter function.

(b) Apply your function to the case where $f(x) = e^x \sin x^2$ and $x = 0, 1, 2$. Also apply your function to the case where $f(x) = 1/(1+9x^2)$ and $x = -1, 0, 1$. Print out or write out your results.

9. (Math 274) Let f be infinitely continuously differentiable. Use Taylor series to find A, B, C, depending on h, such that for all x,

$$f'(x) = Af(x) + Bf(x+h) + Cf(x-2h) + \mathcal{O}(h^p),$$

for the largest p possible.