# Math 174 Midterm 1

October 26, 2016

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit**.

Print Name: ________________________________________________

Student ID: ________________________________________________

Signature and Date: ____________________________________________

| Problem | Score |
|---|---|
| 1 | /25 |
| 2 | /25 |
| 3 | /25 |
| 4 | /25 |
| Total | /100 |

1. (25 pts) (Matlab) Write a **Matlab program** with header:

   function [x] = backsub(n,U,b)

   that inputs $n$; an **upper triangular** $n \times n$ matrix $U$; and an $n \times 1$ vector $b$, and outputs $x$, the solution of $Ux = b$, using **back substitution**. Remember back substitution:

$$x_i = \left( b_i - \sum_{j=i+1}^{n} u_{ij} x_j \right) \Bigg/ u_{ii}.$$

2. (25 pts) Circle **True or False** in each part. You do **not** have to show your work in this problem.

   (a) Let $f$ be a continuous function in $[a, b]$, $a < b$, with more than one root, and suppose $f(a)$ and $f(b)$ have different signs. The bisection method, using starting interval $[a, b]$, will produce a sequence of approximations that **always converges** to the root **closest** to the midpoint of $[a, b]$.

   **True** or **False**

   (b) Let $g$ be a continuous function in $[a, b]$, $a < b$, and suppose $g(a) < a$ and $g(b) > b$. Then $g$ **has to have** a fixed point in $[a, b]$.

   **True** or **False**

   (c) Let $g(x) = 2x^4 - x^3 + 2x^2 - 2x + 1$. Then $p = 1/2$ is a fixed point and fixed point interations will **always converge** to $p$ if $p_0 \neq p$ is chosen **close enough** to $p$.

   **True** or **False**

   (d) Suppose we want a root of $f(x)$. Let $\{p_n\}_{n=0}^{\infty}$ be the sequence of approximations generated by the secant method with $p_0 = a, p_1 = b$, and let $\{\tilde{p}_n\}_{n=0}^{\infty}$ be the sequence of approximations generated by the secant method with $\tilde{p}_0 = b, \tilde{p}_1 = a$. Then $p_k$, if it exists, is **always equal to** $\tilde{p}_k$, for $k = 2, 3, \ldots$. Remember secant method: $p_{n+1} = p_n - f(p_n)(p_n - p_{n-1})/(f(p_n) - f(p_{n-1}))$.

   **True** or **False**

   (e) Suppose we want roots of the continuously differentiable functions $f(x)$ and $\tilde{f}(x) = 5f(x)$. Let $\{p_n\}_{n=0}^{\infty}$ be the sequence of approximations generated by Newton's method on $f(x)$, with $p_0 = a$, and let $\{\tilde{p}_n\}_{n=0}^{\infty}$ be the sequence of approximations generated by Newton's method on $\tilde{f}(x)$, with $\tilde{p}_0 = a$. Then $p_k$, if it exists, is **always equal to** $\tilde{p}_k$, for $k = 1, 2, 3, \ldots$. Remember Newton's method: $p_{n+1} = p_n - f(p_n)/f'(p_n)$.

   **True** or **False**

   (f) Let $f$ be a continuous function in $[-1, 3]$ and suppose $f(-1)$ and $f(3)$ have different signs. Let $\{p_n\}_{n=0}^{\infty}$ be the sequence of approximations generated by the bisection method using starting interval $[-1, 3]$ (so $p_0 = 1$). Then there **has to be** a root $p$ of $f$ satisfying $|p - p_5| \leq 1/20$.

   **True** or **False**

3. (25 pts) Solve the following short problems.

(a) Use the **method of false position**, with starting interval $[2, 3]$, to generate the $p_0$ and $p_1$ approximations to the root of $f(x) = x^2 - 5$. Remember false position:

$$p_n = b_n - f(b_n)\frac{b_n - a_n}{f(b_n) - f(a_n)}.$$

(b) Solve using **Gaussian elimination** without pivoting and **back substitution**, under 2-digit **rounding**, the linear system $A\vec{x} = \vec{b}$, with

$$A = \begin{bmatrix} -3 & 3 \\ 1 & 2 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

4. (25 pts) Let $f$ be a twice continuously differentiable function with root at $p$, and suppose $f'(p) \neq 0$. Let $\{p_n\}_{n=0}^{\infty}$ be the sequence of approximations computed by Newton's method with starting guess $p_0$. If $p_n$ converges to $p$, use Taylor series to show the **order of convergence** is 2, and find the **asymptotic error constant**. Remember Taylor series:

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(c)}{2}(b-a)^2,$$

for some $c$ between $a$ and $b$; and order of convergence:

$$\lim_{n\to\infty} \frac{|p - p_{n+1}|}{|p - p_n|^{\alpha}} = \lambda.$$