Math 174 Midterm 2

November 16, 2016

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit**.

Print Name: ____

Student ID: ____

Signature and Date: ______________________________________

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

1. (25 pts) Write a **Matlab program** with header:

   function [value] = interpolatingpoly(m,x,y,z)

   that inputs number of data points m; x-coordinates of the data points, in the vector x; y-coordinates of the data points, in the vector y; and location z, and outputs $p(z)$, calculated using the **Lagrange form** of the interpolating polynomial p for the data points.

   Use only basic programming, such as for loops and if statements, and do **not** use any of Matlab's vector-vector or matrix-vector operations. Remember Lagrange form for $n+1$ data points $(x_0, y_0), \ldots, (x_n, y_n)$:

$$p(x) = \sum_{i=0}^{n} \left[y_i \left(\prod_{j=0, j \neq i}^{n} \frac{x - x_j}{x_i - x_j} \right) \right].$$

2. (25 pts) Circle **True or False** in each part. You do **not** have to show your work in this problem.

   (a) Let $p(x)$ be the interpolating polynomial for the data points with distinct nodes:

   $$(x_0, f(x_0)), (x_1, f(x_1)), \dots, (x_n, f(x_n))$$

   and suppose the divided difference $f[x_0, x_1, \dots, x_n] = 0$. Then $\deg p$ **has to be** $< n$. Remember $f[x_i] = f(x_i)$, and for $i < j$, $f[x_i, \dots, x_j] = \frac{f[x_{i+1},\dots,x_j] - f[x_i,\dots,x_{j-1}]}{x_j - x_i}$.

   True or **False**

   (b) Let A be an $n \times n$ nonsingular matrix and let P, L, U give the PLU factorization, $PA = LU$, derived from Gaussian elimination with **partial pivoting**. Then, writing $L = (l_{ij})$, $|l_{ij}|$ **is always** ≤ 1, for all $i, j = 1, \dots, n$.

   True or **False**

   (c) Let A be an $n \times n$ lower triangular, nonsingular matrix and let b be an $n \times 1$ vector. Then Gauss-Seidel on the linear system $Ax = b$ **always converges** for any initial guess. Remember Gauss-Seidel: $x^{(k+1)} = (D - L)^{-1} U x^{(k)} + (D - L)^{-1} b$ or $x_i^{(k+1)} = \left[b_i - \sum_{j=0}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right] / a_{ii}$.

   True or **False**

   (d) Let A be an $n \times n$ upper triangular, nonsingular matrix and let b be an $n \times 1$ vector. With $x^{(0)}$, let $x^{(k)}$ be generated using **Jacobi method** on $Ax = b$. With $\tilde{x}^{(0)} = x^{(0)}$, let $\tilde{x}^{(k)}$ be generated using **Gauss-Seidel** on $Ax = b$. Then $\tilde{x}^{(k)}$ has to equal $x^{(k)}$ for any $x^{(0)}$ and for all $k \geq 0$. Remember Jacobi: $x^{(k+1)} = D^{-1}(L + U)x^{(k)} + D^{-1}b$ or $x_i^{(k+1)} = \left[b_i - \sum_{j=0, j \neq i}^{n} a_{ij} x_j^{(k)} \right] / a_{ii}$.

   True or **False**

   (e) Suppose we know $\frac{1}{\sqrt{n}} ||x||_2 \leq ||x||_\infty \leq ||x||_2$, for all $n \times 1$ vectors x. Then $||A||_\infty$ **has to be** $\leq \sqrt{n} ||A||_2$, for all $n \times n$ matrices A. Remember, for natural matrix norms, $||A|| = \max_{x \neq 0} \left(\frac{||Ax||}{||x||} \right)$.

   True or **False**

   (f) Let $f(x) = x^3 + x^2 + x + 1$ and let $p(x)$ be the interpolating polynomial for the data points $(i - 3, f(i - 3)), i = 0, \dots, 6$. Then $p(x)$ **has to equal** $f(x)$ for all x.

   True or **False**

3. (25 pts) Solve the following short problems.

(a) Find **Newton's form** of the interpolating polynomial for the table of data:

x	-2	1	3
$f(x)$	-1	5	4

. Remember, Newton's form

$$p(x) = \sum_{i=0}^{n} \left[f[x_0, \ldots, x_i] \left(\prod_{j=0}^{i-1} (x - x_j) \right) \right].$$

(b) Consider $Ax = b$, where

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 4 & -1 \\ -2 & 2 & -1 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 8 \\ 5 \end{bmatrix}.$$

Use **Jacobi method** with initial guess $x^{(0)} = [3, 2, -5]^T$ to generate $x^{(1)}$.

4. (25 pts) Let $A = \begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$. For a given b, let x be the solution to $Ax = b$, and for a given δb, let $x + \delta x$ solve $A(x + \delta x) = b + \delta b$. Then for a vector norm $||\cdot||$ and its associated natural matrix norm, **prove** the error bound

$$\frac{||\delta x||}{||x||} \leq \kappa(A)\frac{||\delta b||}{||b||}.$$

Then, using $||\cdot||_\infty$, find **one example** of $b \neq 0$ and $\delta b \neq 0$ such that equality in this error bound is achieved. Note δx and δb are names of vectors and **does not** refer to the multiplication of a constant δ to a vector. Remember

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$