Math 100A Test 2 100 points November 21, 2011

Notation: G is a multiplicative group with center Z(G).
S_n denotes the group of n! permutations of the symbols 1, 2, 3, ... , n ,
and A_n denotes its subgroup of even permutations.
D_n is the dihedral group generated by R and F, where the rotation
R has order n and the flip F has order 2.
Points: #**1, 2, 3, 5a, 5b** are worth 16 pts each, # **4** is worth 20 pts.

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(1) Prove in detail that the index $|S_9 : A_9|$ equals 2.

(2) List all elements in D_8 that are conjugate to the flip F in D_8. Justify.

(3) Let G be a non-abelian group.
Prove in detail that the quotient group G/Z(G) cannot be cyclic.

(4) Let G be a cyclic group of order 31. Prove in detail that the "squaring map" f is an automorphism of G, where f is defined by $f(g) = g^2$ for every element g in G.

(5) Let A and B be subgroups of a group G, with B normal in G.
Let C denote the subgroup $A \cap B$.
(a) Prove that C is normal in A.
(b) Prove that the map $f: AB \to A / C$ is well-defined, where f is given by
$f(ab) = aC$ for all a in A and all b in B.

(6) Extra Credit Problem:
Let $\sigma = (1234)(5678)$ in S_8 (note σ is the product of two disjoint 4-cycles).
How many elements of S_8 commute with σ ? Justify.