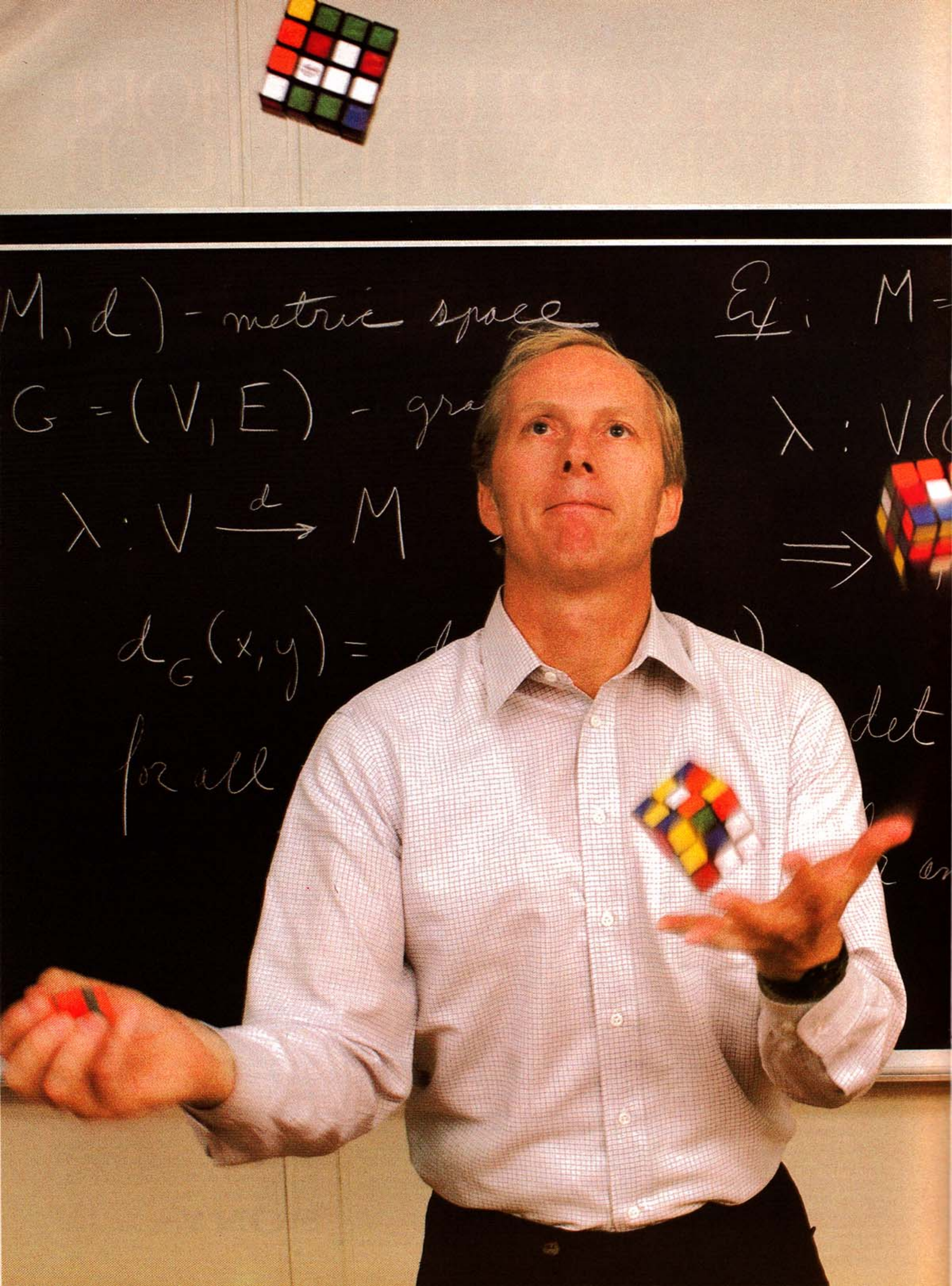
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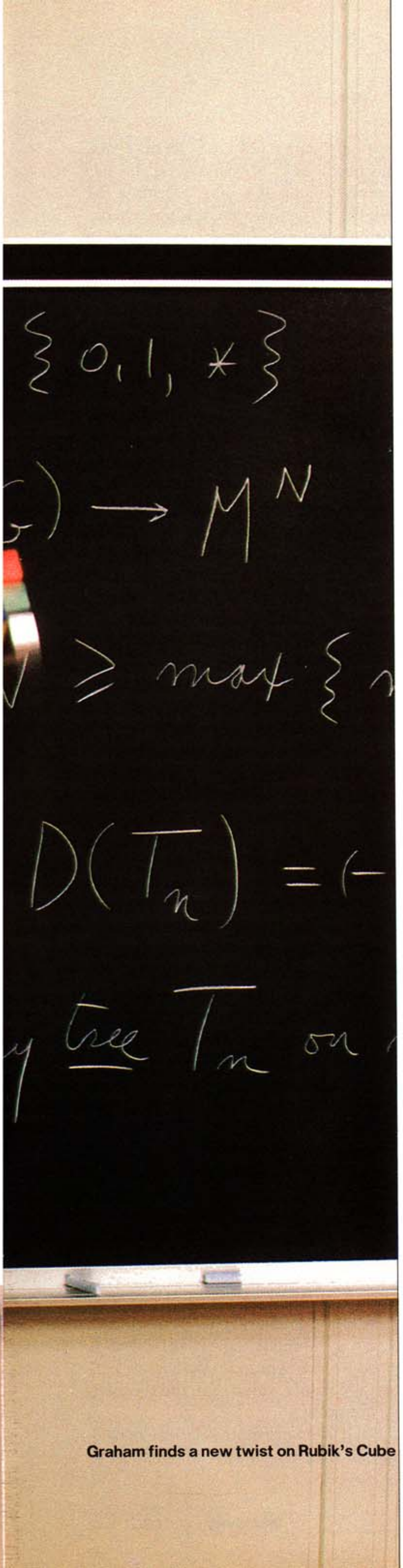

Graham finds a new twist on Rubik's Cube

RONALD GRAHAM:
THE PERIPATETIC NUMBER JUGGLER

A versatile Bell Labs scientist, using pure mathematics, solves practical problems
by BRUCE SCHECHTER

The artistically landscaped buildings contain some of the most advanced scientific apparatus in the world. But in one small room at the Bell Laboratories complex in Murray Hill, New Jersey, the only equipment in sight seems more appropriate to a novelty shop. The room is Ronald Graham's office, and it is cluttered with Rubik's Cubes, geometric puzzles, Chinese illusions, juggling balls, and other curiosities. As Graham chats with a visitor, his hand moves across his desk toward a blue plastic object shaped like a cigar flattened on one side. Catching the visitor's eye, Graham gives the object an offhand flick that sets it spinning clockwise.

After a moment, something strange happens: the spinning object slows, stops, and then—reverses! Graham does an exaggerated double take, his eyebrows shoot up, he shrugs, and he asks with mock surprise, "How did *that* happen?" The answer, he subsequently explains, has something to do with the cigar's subtly asymmetrical shape and some complicated physics and mathematics.

Graham, 46, is the head of Bell Labs' Mathematical Studies Center, and he also delights in mysteries, mystification, and puzzles. One of the world's leading combinatorial mathematicians, he has solved professional problems that, to the initiated, are as puzzling and wonderful as the reversing spin of the blue cigar. For the past 20 years he has confronted the formidable challenges that arise from the need to route hundreds of millions of telephone calls through the intricate communications web of cables, microwaves, and satellites that embraces the earth. The mathematical techniques and theorems he has developed in the process can be applied not only to the routing of information within a computer, but also to the efficient scheduling of an astronaut's day, or even to the allocating of an entire nation's resources. He has the rare ability to translate real-world problems into mathematics, and as colleague Persi Diaconis, a mathematician at Stanford University, puts it, "Ron, as much as anybody, is responsible for bringing high powered math to bear on computer science."

Graham's life is itself a scheduling problem that would try the capacity of any computer. He is a remarkably prolific mathematician, publishing more than a dozen papers a year. He sits on the editorial boards of some 20 mathematics journals, travels extensively, and lectures frequently. He is also a talented and dedicated juggler, and has been honored for his skills by being elected president of the International Jugglers Association. He constantly works at improving his juggling technique; a net hangs from his office ceiling to snare the occasional ball that escapes him. In his younger years he earned money as a trampoline acrobat, and he still stays in shape by bouncing and flipping on his home trampoline.

Even now he delights in finding new skills to master—and he masters them better than most. "You should never be afraid to be a beginner," he says. "It keeps your mind open and flexible." Since reaching adulthood, Graham has learned to bowl (he has rolled a couple of 300 games), throw a boomerang, play Ping-Pong (he was the Bell Labs champion), parachute jump, speak Chi-

PHOTOGRAPHS BY KEN REGAN—CAMERA 5

nese, and play the piano. He has run in a marathon, and is now learning to play tennis. How does he do it? "Well," he says in a slow, quiet voice, "there are a hundred and sixty-eight hours in a week."

Many of those hours are consumed by his job. Besides administering a large department, directing its research, and solving mathematical problems, Graham thinks nothing of dropping everything to help a graduate student untangle a messy proof, or to locate an obscure reference for a colleague down the hall. "I think all of us wish we had more time with him," says Diaconis, "but when you're really in trouble with a mathematical proof or something, he'll never leave you hanging. He's a nice genius."

He also thinks very fast. Recently, Diaconis asked him for advice on a problem involved in producing images from data generated by a brain-scanning apparatus. Three days later Graham returned with 60 pages of calculations that solved the difficulty. Another time, Diaconis was giving a talk describing research he and Graham were conducting, and concluded with "This case is unsolved." At that point, Graham rose from his seat in the audience and outlined a solution he had just thought out. The audience was so impressed it applauded—a rare show of emotion at a meeting of mathematicians.

One reason for Graham's prodigious productivity, he says, is that he is often on the move and difficult to reach. That gives him time for uninterrupted thought. As he explains, "So far there are no telephones on airplanes," and indeed many of his best mathematical ideas emerge while he is belted into an airplane seat. Furthermore, traveling keeps him in touch with almost everything that is happening in his field of mathematics. If a mathematician at Stanford has a problem, Graham is likely to know of some mathematician on the other side of the country who has a solution. He serves as a mathematical matchmaker, bringing together people and problems.

In the jargon of mathematicians, the problems that Graham specializes in are known as "hard"—because of the mind-boggling complexity they assume in successive steps. An example is the infamous traveling salesman problem: a salesman must pick the shortest possible route for his visits to various cities. If there are only a few cities on his route, the problem is almost trivial; he has merely to look at a map, calculate the lengths of possible routes, and choose the shortest one. But as the number of cities increases, the problem quickly grows beyond the capacity of even the fastest computers.

No general and perfect solution has ever been found for the traveling salesman problem, but mathematicians like Graham keep working to improve the partial solutions they have discovered. The results of their labors have practical applications in scheduling, economics, cryptography, and computer science, all of which involve "hard problems," technically described as "non-deterministic polynomial complete" problems. But the hard problem of greatest interest to Graham's employer is the efficient routing of telephone messages between tens of thousands of communities—a problem for which a bad solution could cost untold millions of dollars. By finding better approaches to the traveling salesman and other, related problems, Graham and his colleagues continue to improve long-line telephone efficiency, thereby earning their salaries many times over.

At times Graham's special gifts have helped other organizations, including NASA. During the Apollo moon program, the space agency needed to evaluate mission schedules so that the three astronauts aboard a spacecraft could find the time to perform all necessary tasks—experiments, eating and sleeping, and managing their vehicle. The number of ways to allot these tasks was astronomical—too vast even for a computer to sort out—and NASA officials were concerned about wasting valuable time through inefficient scheduling. NASA had based its schedules on mathematical techniques that produced good but not perfect solutions. The agency wanted to know just how badly it might be erring, so it turned to Graham, who had pioneered a field of mathematics known as worst case analysis. Graham provided a reassuring answer; he proved that NASA's methods never produced a schedule that was more than a few percentage points worse than the theoretical optimum.

But it is in pure mathematics—mathematics with no obvious application to

Graham and David Hagelbarger mass-produce Penrose tiles at Bell Labs

telephone lines or spaceships—that Graham has earned his distinguished reputation. His contributions have been mainly in a field known as Ramsey theory, named for Frank Ramsey, a brilliant mathematician who, before his death in 1930 at the age of 26, published a paper that was to influence a generation of mathematicians. As Graham explains it, "Ramsey theory says that complete disorder is impossible. There is always structure somewhere." Since, according to Graham, mathematics is the study of order, Ramsey theory is particularly appealing to him—so much so that the license plate on his car reads RAMSEY.

A simple prediction of Ramsey theory can be stated in human terms. If two people are selected at random from any group, they will either know each other or be strangers. But Ramsey theory shows that in any group of six or more people, three will either all know one another or all be strangers. This fact is fairly easy for mathematicians to prove, but as the numbers grow larger Ramsey theory becomes more difficult to apply. For instance, it can be shown that in a group of 18 people there must be four who all know each other or are all strangers. But how big a group is required so that it will always include five mutual friends or five mutual strangers? Nobody yet knows.

Graham defies gravity with Marc and Cheryl

Graham's research has increased the scope of Ramsey theory well beyond this type of problem, and it was for this work that in 1972 he shared, along with Bruce Rothschild and Klaus Leeb, the prestigious Polya Prize, awarded by the Society for Industrial and Applied Mathematics. His work in Ramsey theory has also been recognized by no less an authority than the *Guinness Book of World Records*, which acknowledges that Graham holds the record for identifying the largest number ever used in a mathematical proof. The number is so big that it has to be written using a special notation, developed by Stanford computer scientist Donald Knuth, in which exponentials pile upon exponentials in dizzying, astronomically large towers of powers.

Diaconis believes that Graham's work on Ramsey theory "will take a hundred years to be applied, if it ever is." But Graham is convinced that all mathematics is related to the real world, that math really "exists" in some sense. "Is mathematics a creation of the human mind," he asks, "or is it really out there? I happen to believe that it is there, it was always there, and would be there without us. Our job is to discover, not to create. But the act of discovery is a creative process." A field of mathematics known as Riemannian geometry, the geometry of curved space, was investigated, as Graham points out, by 19th century mathematicians who had no notion of applying it to anything "real." But years later Einstein used Riemannian geometry as one of the cornerstones of his theory of general relativity. Concludes Graham, "From my point of view, in some sense, the basic reality is mathematics."

Graham's approach to solving mathematical problems is similar, he says, to his approach to juggling or learning any other new skill: systematic, determined, and intuitive. As he explains, "I may have abstracted this essential feature more than a lot of other people have: trying to break a large problem into smaller problems that are a little easier to deal with." It is like learning Chinese, he says, first mastering the individual sounds, and only then moving on to words and sentences.

Graham also tries to instill these principles when he teaches, and he has taught many subjects, including all levels of mathematics, gymnastics, and juggling. "In order to teach," he says, "you have to be able to focus on exactly where the difficulty is, and that means splitting the problem apart until you can really locate exactly what it is that is difficult." Still, the basic unpredictability of mathematics, Graham concedes, makes it harder to teach than juggling.

It is this unpredictability, he says,

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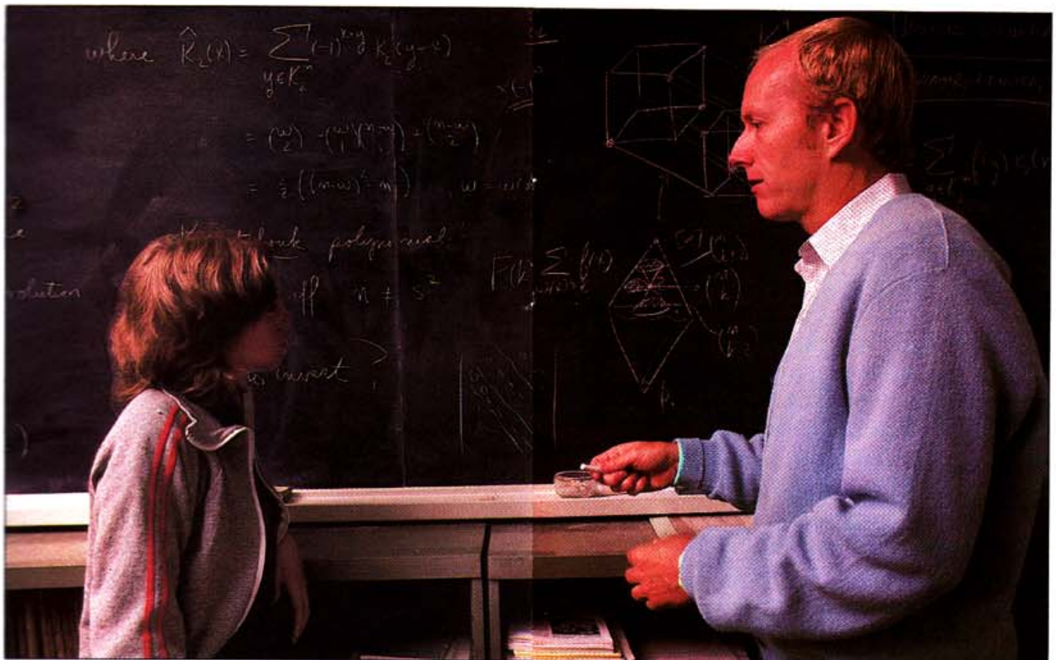


Helping a student prove a tricky theorem

that often causes the systematic, analytic approach to break down. "There's really more insight than mechanics in mathematics, more leaps of intuition, more making of connections," and, he says, many of the best mathematical proofs seem to start out with a *non sequitur*, in which the mind jumps from the main stream of thought to something seemingly unrelated that ultimately provides the key to the problem. "I feel strongly that your brain is working on many channels at the same time. In a way, it's a kind of mental juggling. A lot of things going on at once. Even when I'm talking, when I'm sleeping, eating, or on the trampolin, part of my brain is dedicated to trying and experimenting, piecing things together." This tangential type of thinking is reflected in Graham's style of speaking; the words come out like accumulating pieces of a puzzle that grows in what sometimes appears to be haphazard patterns.

Graham acknowledges that some problems defy his preferred approach and call for outright wrath. "You can start to get mad at a problem," he says. "Did you ever see a Daffy Duck cartoon? When a few bad things happen to Daffy—Elmer Fudd or somebody blows him up in a number of different ways—he says, 'Of course, you know this means war!' And you often get a very appealing, attractive problem that gets under your skin. You're living the problem, it's part of you, it's always in the background, running. In some sense this means war. It's kind of life and death."

Graham's wars rage on for a week or so at a time, but then he must back off. As he explains, using words drawn from the vocabulary of mathematics, "It isn't always optimal for me to stay in this mode for weeks and weeks." When a problem *really* gets under his skin, Graham will occasionally resort to putting out a contract on the problem by offering a cash prize for its solution. "It's putting your money where your mouth is," he says. "You say, 'Look, I'd be happy to pay a hundred dollars to have somebody solve this thing and get it out of my hair and relieve my misery. Either prove it or show me that I'm wrong, but let me know!' "

Failures sometimes lead Graham to doubt mankind's fitness to do mathematics. "Somehow I feel that human beings and the human brain are relatively recent developments, not designed to prove the Riemann hypothesis or study the space-time structure of the universe," he muses. "Our brains were designed to keep us out of the rain, pick berries, and keep us from being killed. So the brain did that, but now it's got a whole new set of challenges—and we're getting better, but we're still a long way from being good at them."

Graham began to excel in mathematics almost from the start. Born in Taft, California, a small town northwest of Los Angeles, he spent most of his youth moving back and forth between Georgia and California as his father kept changing jobs, moving from oil fields to shipyards. Graham never went to the same school for more than a year and a half. He managed to acquire and shed a Southern accent alternately as he repeatedly crossed the country. "Because I was always kind of a new kid," he recalls, "I was never really accepted socially into the in-groups." Also, because he skipped a few grades, he was always younger and smaller than his classmates. "I was of negative value to the team, so I didn't get involved."

Instead, Graham became interested in mathematics and astronomy. The stars have always fascinated him, but, largely because of a few good teachers and the fact that "math is portable," he fixed on mathematics. In the fifth grade, a teacher taught him how to extract square roots of numbers, and with a mathematician's gift for generalization, he wondered whether he could extend the technique to finding cube roots. It was not until much later that he worked out a way to do it, but thinking about the problem led him to higher mathematics.

Eventually Graham's parents were divorced, and he moved to Florida with his mother. Alert to scholarship possibilities, she signed him up to try for a Ford Foundation scholarship. He did well on the examination, got the scholarship, and at 15, without graduating from high school, he enrolled at the University of Chicago.

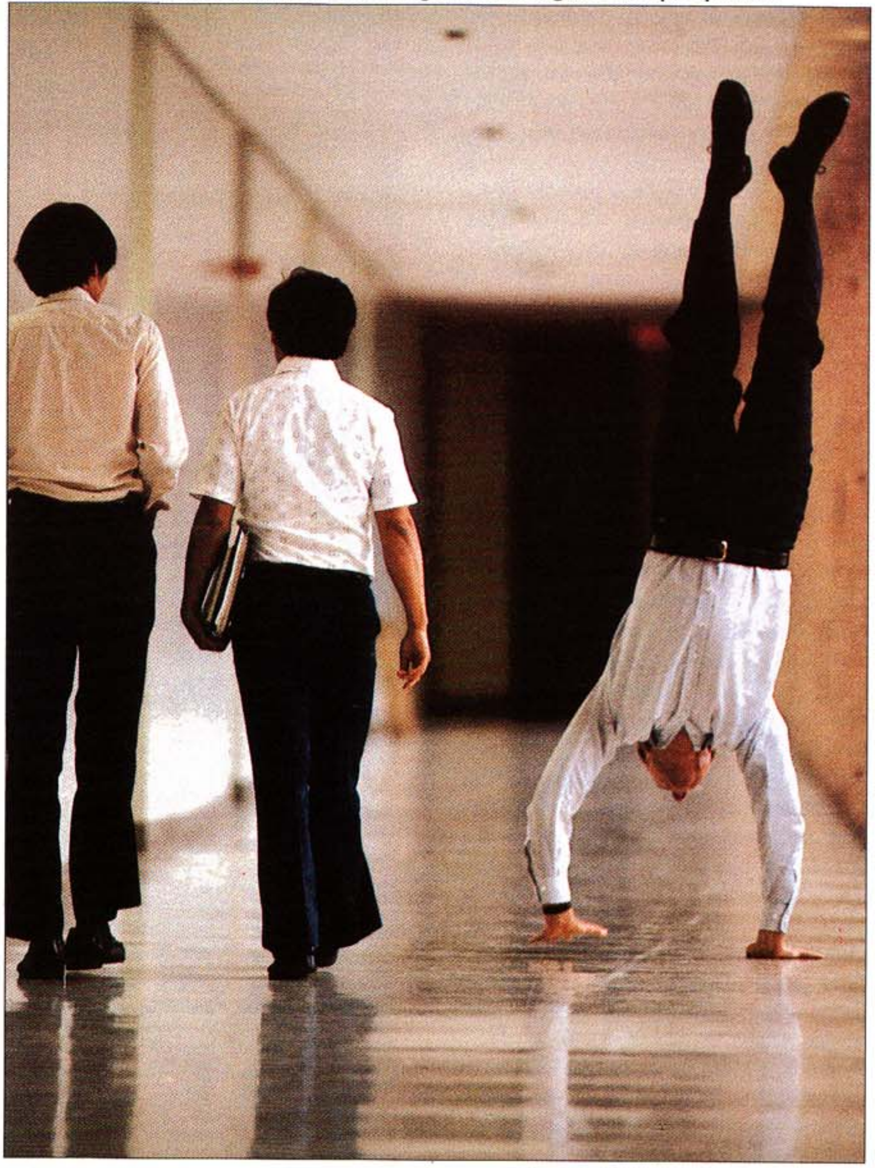
Finally among other bright young students from all over the country, Graham flourished. Still small for his age—though he would eventually reach 6′ 2″—he began to learn gymnastics, a sport in which the short are not at a disadvantage. He went on to master juggling and the trampoline, and for the first time found himself admired for talents other than his obvious intelligence. Juggling also gave Graham a new skill that he could carry anywhere and that would give him instant acceptance—at least by some people. "If you see another juggler juggle five balls, you already have a certain tie to this person," he says. "You know that he took the time, the energy. You know that what he's going through you went through yourself. There's a certain feeling of comradeship, and it's just as true in mathematics."

When his scholarship ran out, he transferred to Berkeley for a while, and then, facing the draft, enlisted in the Air Force, where he thought he could work hard and get a choice assignment. He did work hard, finishing at the top of his class in communications, but he found himself posted to Fairbanks, Alaska, which he did not regard as choice. Undeterred, he signed up at the University of Alaska by day and worked at his Air Force unit at night. Although he majored in mathematics, the university was not accredited in math, and he was awarded a degree in physics instead.

Serving out his enlistment, he returned to Berkeley to earn a Ph.D. While he was there, he and two other students formed a professional trampoline troupe, earning money by performing at schools, supermarket openings, and even the circus. He also met and married Nancy Young, a brilliant fellow math major. In 1962, with a fresh Ph.D., he joined the staff of Bell Labs.

Graham moved to New Jersey with Nancy, where they had two children—Cheryl, now 20 and a journalism major at Northwestern University, and Marc, 15, a tenth-grader. Neither Marc nor Cheryl wants to follow in their father's mathematical footsteps, but both are expert trampolinists and competent jugglers. Their father has offered them a prize of $25 for juggling three balls, $100 for juggling four balls, and a whopping $1,000 for juggling five balls. So far both have collected $25. Unlike the mathematical prizes that Graham offers, the juggling prizes are good only within his family.

Stuck on a problem, Graham leaves nothing unturned to gain a new perspective



Divorced four years ago, Graham has since lived alone in a small house about two miles from his office. There, the walls are decorated with interlocking patterns of Penrose tiles (named for their British inventor, the mathematical physicist Roger Penrose)—small plastic panels, shaped like kites and arrowheads, that can fit together in an infinite number of ways. Graham became so fascinated with Penrose tiles that he and a colleague made a plastic mold to mass-produce them. Of the 100,000 he has made, many have been sent to mathematicians around the world for use as both playthings and devices to help prove interesting new theorems.

Gadgets are everywhere in Graham's house. He owns a bewildering array of calculators, as well as several shortwave receivers and two dozen Rubik's Cubes (he is one of the world's leading experts on the cube and a consultant to the manufacturer on copyright-infringement cases). Other belongings include a computer chess game, a juggling apparatus, a video tape recorder, and many more electronic gadgets.

When he needs reassurance about his life and his profession, Graham thinks about Gödel's theorem, which states, roughly, that there is no end to mathematics, that the adventure of mathematical discovery will continue forever. "Mathematics is to me, and to a lot of mathematicians, a very exciting thing," Graham says. "It's an open-ended challenge. No one's good enough to do even a small fraction of what there is to be done. The problems are more than adequate to challenge anyone, and as far as I can tell, that's always going to be the case. It's like juggling. When have you become the absolute juggler? When you can do all the tricks? Well, there's always one more ball." ▣