Retrospective
In
Combinatorics:
Honoring
STANLEY'S 60th
birth-
Day

June 22–26, 2004
Massachusetts Institute of Technology

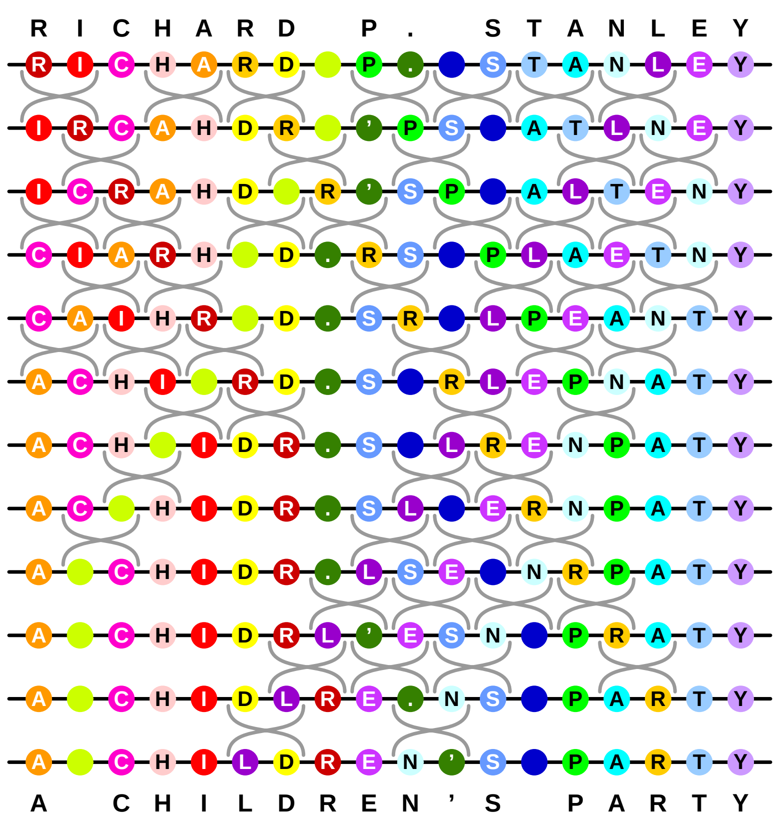
Abstract and schedule booklet

Organizing committee
Ira Gessel
Bruce Sagan, John Stembridge, Vic Reiner, David Ellwood
Igor Pak, Tim Chow, Glenn Tesler

In partnership with
Massachusetts Institute of Technology, Dept. of Mathematics

Clay Mathematics Institute

National Science Foundation



Organizing committee

Ira Gessel	Brandeis University
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John Stembridge	University of Michigan
Vic Reiner	University of Minnesota
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Glenn Tesler	University of California, San Diego

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Massachusetts Institute of Technology, Department of Mathematics
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The organizing committee would also like to thank Conan Leung (University of Minnesota) for making the conference poster. Photo provided by Richard Stanley. Anagram graphics provided by Glenn Tesler. Clay Math Institute logo provided by Clay Math Institute.

Finally, the organizing committee thanks Sara Billey for organizing the ping-pong tournament.

Festschrift

The *Stanley Festschrift* is a special issue of the Electronic Journal of Combinatorics, with guest editor Bruce Sagan. Papers may be submitted to the EJC managing editor (not Bruce Sagan) for refereeing from now through June 30, 2005, with a note that they are for the *Stanley Festschrift*. Accepted papers will be published electronically throughout that period. Several seed articles have already been accepted and appeared. All appropriate submissions will be considered, not just those from conference speakers and attendees. The special issue and submission instructions are available from the main Electronic Journal of Combinatorics website at `http://www.combinatorics.org` by clicking on the link for Volume 11(2).

Conference program

Tuesday, June 22 (room 54-100)

8:00 AM		**Registration and continental breakfast**
9:00 AM	David Vogan	*Opening remarks*
9:15 AM	Jim Propp	*Richard Stanley and combinatorial reciprocity*
9:45 AM	Bruce Sagan	*Monomial Bases for NBC Complexes*
10:15 AM		**Coffee break**
11:00 AM	Tom Roby	*The Lattice of Threshold Graphs and Split Graphs*
11:30 AM	David Wagner	*Remarks on one combinatorial application of the Aleksandrov-Fenchel inequalities*
NOON		**Lunch break**
2:00 PM	Igor Pak	*Rigidity and polynomial invariants of convex polytopes*
2:30 PM	Timothy Y. Chow	*Perfect matching conjectures and their relationship to* $P \neq NP$
3:00 PM	Herbert S. Wilf	*The method of characteristics, and a conjecture of Graham, Knuth and Patashnik*
3:30 PM		**Coffee break**
4:00 PM	Ira Gessel	*P-Partitions and Stirling Numbers*
4:30 PM	Curtis Greene	*Words avoiding a reflexive acyclic relation*

Wednesday, June 23 (room 54-100)

8:00 AM		**Registration and continental breakfast**
9:00 AM	William Y. C. Chen	*Labelling Schemes for 2-Motzkin Paths*
9:30 AM	Robert A. Proctor	*d-Complete posets generalize Young diagrams for the hook product formula*
10:00 AM	David J. Grabiner	*Asymptotics for the Distributions of Subtableaux in Young and Up-Down Tableaux*
10:30 AM		**Coffee break**
11:00 AM	Sara Billey	*Intersecting Schubert Varieties*
11:30 AM	Takayuki Hibi	*Commutative Algebra after Richard Stanley*
NOON		**Lunch break**
2:00 PM	Mark Skandera	*Schur nonnegativity and the Bruhat order*
2:30 PM	John Stembridge	*Tight quotients and double quotients in the Bruhat order*
3:00 PM		**Coffee break**
4:00 PM	Art Duval	*A Relative Laplacian spectral recursion*
4:30 PM	Victor Reiner	*Reciprocal domains and Cohen-Macaulay d-complexes in* $\mathbf{R}^d$
6:30 PM		**Banquet at Marriott in Kendall Square**

Thursday, June 24 (room 54-100)

8:00 AM		**Registration and continental breakfast**
9:00 AM	Glenn Tesler	*Reconstructing the Genomic Architecture of Ancestral Mammals*
9:30 AM	Federico Ardila	*The Bergman complex of a matroid and phylogenetic trees*
10:00 AM	Caroline Klivans	*The Positive Bergman Complex of an Oriented Matroid*
10:30 AM		**Coffee break**
11:00 AM	Adriano M. Garsia	*On the m-quasi-invariants of* S_3
11:30 AM	Peter McNamara	*Cylindric Schur Functions*
NOON		**Lunch break**
2:00 PM	Gabor Hetyei	*Orthogonal polynomials represented by CW-spheres*
2:30 PM	Louis J. Billera	*CW-spheres from meet-distributive lattices*

Friday, June 25 (room *34-101*)

8:00 AM		**Registration and continental breakfast**
9:00 AM	Bo-Yin Yang	*A Study in XL*
9:30 AM	Emden R. Gansner	*How to Draw a Graph*
10:00 AM	Karen L. Collins	*Graph Homomorphisms and Graph Cores*
10:30 AM		**Coffee break**
11:00 AM	Julian West	*Perfect matching graphs for Gale-Robinson sequences*
11:30 AM	Francesco Brenti	*Special matchings and Kazhdan-Lusztig polynomials*
Noon		**Lunch break**
2:00 PM	Neil L. White	*Lagrangian Matroids*
2:30 PM	Patricia Hersh	*Chains of modular elements*
3:00 PM	Anders Björner	*Geometrically constructed homology bases for certain partition lattices*
4:00 PM		**Ping-pong tournament (in Dupont)**

Saturday, June 26 (room 54-100)

8:00 AM		**Registration and continental breakfast**
9:00 AM	Dominique Foata	*Word and permutation statistics*
9:30 AM	Einar Steingrímsson	*Decreasing subsequences in permutations and Wilf equivalence for involutions*
10:00 AM	Miklos Bona	*Why one pattern is easier to avoid than another one*
10:30 AM		**Coffee break**
11:00 AM	Alexander Postnikov	*Permutohedra, associahedra, and beyond*
11:30 AM	Sergey Fomin	*Cluster algebras of classical type*
Noon		**Lunch break**
2:00 PM	Christos Athanasiadis	*Ehrhart polynomials, simplicial polytopes and magic squares*
2:30 PM	Lynne Butler	*On the area of cyclic polygons*
3:00 PM		**Coffee break**
4:00 PM	Daniel J. Kleitman	*Partitioning a Planar Graph into a Forest and Something Else*
4:30 PM	Richard Stanley	*Problems I couldn't solve*
5:00 PM		**Adjourn**

Speaker index

Abstracts

Tuesday, June 22

8:00 AM **Registration and continental breakfast**

9:00 AM **David Vogan (Massachusetts Institute of Technology)**
Opening remarks

9:15 AM **Jim Propp (University of Wisconsin)**
Richard Stanley and combinatorial reciprocity
In this talk I will sketch a point of view that ties together two seemingly unrelated results of Stanley's: his result on the value of the characteristic polynomial of a graph when the number-of-colors variable is set equal to a negative integer, and his theorem about the palindromicity of the coefficients of polynomials involved with counting domino tilings of rectangles.

9:45 AM **Bruce Sagan (Michigan State University)**
Monomial Bases for NBC Complexes
In a seminal paper, Richard Stanley noted that the Stanley-Reisner ring R of an NBC (no broken circuit) complex of a graph G is Cohen-Macaulay. Jason Brown gave an explicit description of a homogeneous system of parameters for R in terms of fundamental edge-cuts in G. So R modulo this h.s.o.p. is a finite dimensional vector space. We conjecture an explicit monomial basis for this vector space in terms of the circuts of G and prove that the conjecture is true for several families of graphs. This is joint work with Jason Brown (Dalhousie University).

10:15 AM **Coffee break**

11:00 AM **Tom Roby (Cal State Hayward)**
The Lattice of Threshold Graphs and Split Graphs
To any graph we may associate the sequence of degrees of its vertices. Arranging these in nonincreasing order gives an integer partition of twice the number of edges. We call such a partition *graphic*. A *threshold* graph is one whose degree sequence is maximal (in the dominance order) among graphic partitions. (There are many equivalent definitions.)
By exploiting the interplay between threshold graphs and partitions, we construct a lattice of threshold graphs which is isomorphic to a the lattice of *shifted shapes* (i.e., partitions into distinct parts). We explore the graph theoretic structure of this lattice.
A *split graph* is one whose vertex set can be partitioned as the disjoint union of an independent set and a clique (either of which may be empty). Their degree sequences lie just below the threshold graphs in the poset of graphic partitions. This is joint work with Russell Merris (Cal State Hayward).

11:30 AM **David Wagner (U. Waterloo)**
Remarks on one combinatorial application of the Aleksandrov-Fenchel inequalities
In 1981, Stanley used the Aleksandrov-Fenchel inequalities to prove a logarithmic concavity result enumerating bases of a regular matroid with respect to their size of intersection with a fixed subset of elements. This phenomenon has a central role in the generalization of electrical network theory from graphs to more general matroids. In this context we find an extension of Stanley's theorem with a reasonably short and self-contained proof. This is joint work with YoungBin Choe (Pohang U.).

NOON **Lunch break**

2:00 PM **Igor Pak (MIT)**

Rigidity and polynomial invariants of convex polytopes

I will outline our proof of the Robbins Conjecture on the degrees of generalized Heron polynomials. This is joint work with Maksym Fedorchuk.

2:30 PM **Timothy Y. Chow**

Perfect matching conjectures and their relationship to $P \neq NP$

We conjecture that a distance degree regular graph (one in which the number of vertices at a distance d from a vertex v depends only on d and not on v) has a perfect matching if and only if every connected component has an odd number of vertices. This perfect matching conjecture (among others) is motivated by an old open problem in finite model theory: Does there exist a computable list of polynomial-time Turing machines that comprises all isomorphism-invariant, polynomial-time computable properties of graphs? A negative answer would imply $P \neq NP$. The connection between these problems will be explained in the talk. No background in model theory or logic is needed; my intent is to tempt more combinatorialists into attacking the many purely combinatorial questions that arise in this field.

3:00 PM **Herbert S. Wilf (University of Pennsylvania)**

The method of characteristics, and a conjecture of Graham, Knuth and Patashnik

The method of characteristics is standard for the solution of linear pde's, especially of the first order. We will discuss the method, then give an easy application to a combinatorial problem arising in mathematical biology. Finally, we use the method to find a solution to a "Research Problem" of Graham, Knuth and Patashnik. The form of this solution is quite explicit in an important special case, but rather opaque in general.

3:30 PM **Coffee break**

4:00 PM **Ira Gessel (Brandeis University)**

P-Partitions and Stirling Numbers

Let $S(m,n)$ denote the Stirling number of the second kind. Then for fixed $k > 0$, $S(n+k,n)$ is a polynomial in n of degree $2k$ that vanishes for $k = 0, -1, -2, \ldots, -k$, so there exist polynomials $B_k(t) = \sum_{i=1}^{k} B_{k,i} t^i$ for which

$$\sum_{n=0}^{\infty} S(n+k,n)t^n = \frac{B_k(t)}{(1-t)^{2k+1}}.$$

Richard Stanley and I [*Stirling polynomials*, J. Combinatorial Theory Ser. A 24 (1978), 24–33] showed that $B_{k,i}$ is the number of permutations $a_1 a_2 \cdots a_{2k}$ of the multiset $\{1,1,2,2,\ldots,k,k\}$, with i descents, such that if $u < v < w$ and $a_u = a_w$, then $a_v \geq a_u$. In my talk, I will explain how this result and some generalizations can be proved using Stanley's theory of P-partitions.

4:30 PM **Curtis Greene**

Words avoiding a reflexive acyclic relation

We consider several problems associated with the multivariate generating series $F_{\mathbf{A}}(\mathbf{x}) = \sum_w \mathbf{x}^w$, where the sum is over all words w in an alphabet X whose consecutive letters avoid a reflexive acyclic relation $\mathbf{A}$ on X. This is joint work with Ian Goulden and John Dollhopf.

Wednesday, June 23

8:00 AM **Registration and continental breakfast**

9:00 AM **William Y. C. Chen (Nankai University)**

Labelling Schemes for 2-Motzkin Paths

We give two labelling schemes for 2-Motzkin paths. For each labelling, we may transform a 2-Motzkin path into a partition with certain restriction. Then we construct bijections between a class of restricted 2-Motzkin paths and two classes of restricted partitions. These structures may serve as combinatorial descriptions for the discrete continuity between Motzkin numbers and Catalan numbers. This problem was proposed by E. Barcucci, A.D. Lungo, E. Pergola and R. Pinzani. We may apply the labelling schemes to the problem of counting 2-regular *abba*-free partitions, which are related to the enumeration of directed animals as observed by M. Klazar. We establish a one-to-one correspondence between 2-regular *abba*-free partitions and connected *abba*-free partitions. This is joint work with Eva Y. P. Deng (Nankai University) and Rosena R. X. Du (Nankai University).

9:30 AM **Robert A. Proctor (University of North Carolina)**

d-Complete posets generalize Young diagrams for the hook product formula

Dale Peterson supplied an algebraic geometric proof of the middle "sum = product" step needed for the proof of the result described in this talk. Let P be a poset, and let $e(P)$ be the number of extensions of P to a total order. By 1969, both Richard Stanley and Donald Knuth had asked if there exist other families of connected posets besides shapes, shifted shapes, and rooted trees which possess hook product formulas for $e(P)$. In his thesis, Stanley introduced P-partition generating functions. He obtained a hook product formula for the P-partition generating function of a shape. d-Complete posets arose in the work of this author during the combinatorialization of a representation theoretic basis result of Lakshmibai and Seshadri. Equivalent structures arise in other ways in the work of Peterson and Stembridge. d-Complete posets are mutual generalizations of shapes, shifted shapes, rooted trees, and minuscule (the known Gaussian) posets. Combining the basis result of Laksmibai and Seshadri and Peterson's algebraic geometric result with further steps yields a hook product formula for d-complete posets. This formula generalizes Stanley's and Gansner's formulas for shapes and shifted shapes. (Actually, a more general multivariate version arises.) d-Complete posets account for most of the small connected posets which possess product expressions for their P-partition generating functions.

10:00 AM **David J. Grabiner (National Security Agency)**

Asymptotics for the Distributions of Subtableaux in Young and Up-Down Tableaux

Let μ be a partition of k, and T a standard Young tableau of shape μ. McKay, Morse, and Wilf show that the probability a randomly chosen Young tableau of N cells contains T as a subtableau is asymptotic to $f^\mu/k!$, where f^μ is the number of all tableaux of shape μ. We use a random-walk argument to find the analogous asymptotic probability for randomly chosen Young tableaux with at most n rows; as n goes to infinity, the probabilities approach $f^\mu/k!$ as expected. Our result generalizes to an analogous result for up-down tableaux.

10:30 AM **Coffee break**

11:00 AM **Sara Billey (University of Washington)**
Intersecting Schubert Varieties
Using a blend of combinatorics and geometry, we give an algorithm for finding all flags in any zero dimensional intersection of Schubert varieties with respect to three transverse flags. In particular, the number of flags in a triple intersection is also a structure constant for products of Schubert cycles in the cohomology ring of the flag manifold. Our algorithm is based on solving a limited number of determinantal equations for each intersection. These equations are also sufficient for computing Galois groups and monodromy of intersections of Schubert varieties. We are able to limit the number of equations by using the structure of *permutation arrays* as defined by Eriksson and Linusson. We show that there exists a unique permutation array corresponding to each realizable Schubert problem and give a simple recurrence relation to compute the corresponding rank table. We also give two interesting counterexamples to the Realizability Conjecture of Eriksson and Linusson in using 4 flags in $\mathbb{R}^4$ and Pappus's Hexagon Theorem along with 9 flags in $\mathbb{R}^3$. This is joint work with Ravi Vakil (Stanford University).

11:30 AM **Takayuki Hibi (Osaka University)**
Commutative Algebra after Richard Stanley
The pioneering work of Richard Stanley on the upper bound conjecture in 1975 originated a new branch of commutative algebra, viz., "Combinatorics and Commutative Algebra." My talk will survey recent topics on squarefree monomial ideals related with combinatorics.

NOON **Lunch break**

2:00 PM **Mark Skandera (Dartmouth)**
Schur nonnegativity and the Bruhat order
We define a polynomial $p(x)$ in n^2 variables $x = (x_{1,1}, \ldots, x_{n,n})$ to be *Schur nonnegative* if for each $n \times n$ Jacobi-Trudi matrix A, the symmetric function $p(A)$ is equal to a nonnegative linear combination of Schur functions. We will discuss families of polynomials which are known or conjectured to have the Schur nonnegativity property, and will show that the Bruhat order on S_n may be characterized in terms of this property. This is joint work with Brian Drake (Brandeis) and Sean Gerrish (U. Michigan).

2:30 PM **John Stembridge (U. of Michigan)**
Tight quotients and double quotients in the Bruhat order
It is a well-known theorem of Deodhar that the Bruhat order of a Coxeter group is the intersection of its projections onto maximal parabolic subgroups. As a corollary, one may also obtain the Bruhat order as the intersection of a larger number of simpler quotients by projecting onto maximal two-sided parabolic quotients. In some cases, the quotients, either one or two sided, have an especially nice (i.e., "tight") coordinatization, and this leads to especially simple ways to test the order relation.
We will survey some of the interesting features of these partial orders, primarily for finite and affine Weyl groups. In particular, we will discuss the classification of tight quotients in these cases.

3:00 PM **Coffee break**

4:00 PM **Art Duval (University of Texas at El Paso)**

A Relative Laplacian spectral recursion

The Laplacian spectral recursion, satisfied by matroid complexes and shifted complexes, expresses the eigenvalues of the combinatorial Laplacian of a simplicial complex K in terms of the eigenvalues of $K-e$, K/e, and $(K-e, K/e)$, which are, respectively, the deletion with respect to a ground element e, the contraction with respect to e, and the relative simplicial complex of the deletion modulo the contraction.

I conjecture that this recursion may be generalized to relative pairs of matroid complexes and shifted complexes. This recursion (conjecturally) expresses the eigenvalues of a relative pair R in terms of $R-e$, R/e, $(R-e, R/e)$, and $(R/e, R-e)$, once these expressions are suitably defined for relative pairs.

4:30 PM **Victor Reiner (School of Math., Univ. of Minnesota)**

Reciprocal domains and Cohen-Macaulay d-complexes in $\mathbf{R}^d$

We extend a reciprocity theorem of Stanley about enumeration of integer points in polyhedral cones when one exchanges strict and weak inequalities. The proof highlights the roles played by Cohen-Macaulayness and canonical modules.

The extension raises the issue of whether a Cohen-Macaulay complex of dimension d embedded piecewise-linearly in $\mathbf{R}^d$ is necessarily a d-ball. This is observed to be true for $d \leq 3$, but false for $d = 4$ This is joint work with Ezra Miller (School of Math., Univ. of Minnesota).

6:30 PM **Banquet at Marriott in Kendall Square**

Thursday, June 24

8:00 AM **Registration and continental breakfast**

9:00 AM **Glenn Tesler (University of California, San Diego)**

Reconstructing the Genomic Architecture of Ancestral Mammals

Our recent analysis of genome rearrangements in the human and mouse genomes revealed evidence for a larger number of rearrangements than previously thought. However, two-way analysis of rearrangements cannot reveal the genomic architecture of the ancestral mammals or assign rearrangement events to different lineages. We have now added the rat genome, which greatly enhances the rearrangement analysis and allows us to reconstruct the putative genomic architecture of the ancestral murid rodent genome. In particular, three-way analysis of rearrangements within certain regions, such as the X chromosome, leads to a reliable reconstruction of the genomic architecture of the murid ancestor of these regions and, for the first time, allows one to assign the major rearrangement events to one of human, mouse, and rat lineages. This analysis makes use of many items from algebraic combinatorics including signed permutations, partially ordered sets, and graph theory. This is joint work with Guillaume Bourque (University of Montreal) and Pavel Pevzner (University of California, San Diego).

9:30 AM **Federico Ardila (MSRI)**

The Bergman complex of a matroid and phylogenetic trees

We study the Bergman complex $B(M)$ of a matroid M: a polyhedral complex which arises in algebraic geometry, but which we describe purely combinatorially. We prove that a natural subdivision of $B(M)$ is a geometric realization of the order complex of L_M, the lattice of flats of M. We also show that the Bergman complex $B(K_n)$ of the graphical matroid of the complete graph K_n is homeomorphic to the Whitehouse complex T_n: a well-studied simplicial complex, which describes the combinatorics of the space of phylogenetic trees. This provides a simple explanation of the known result that T_n is homotopy equivalent to a wedge of $(n-1)!$ $(n-3)$-dimensional spheres. This is joint work with Carly Klivans (Cornell University).

10:00 AM **Caroline Klivans (Cornell University)**

The Positive Bergman Complex of an Oriented Matroid

We study the positive Bergman complex $\mathcal{B}^+(M)$ of an oriented matroid M, which is a certain subcomplex of the Bergman complex $\mathcal{B}(\underline{M})$ of the underlying unoriented matroid $\underline{M}$. The positive Bergman complex is defined so that given a linear ideal I with associated oriented matroid M_I, the positive tropical variety associated to I is equal to the fan over $\mathcal{B}^+(M_I)$. Our main result is that a certain "fine" subdivision of $\mathcal{B}^+(M)$ is a geometric realization of the order complex of the proper part of the Las Vergnas face lattice of M. It follows that $\mathcal{B}^+(M)$ is homeomorphic to a sphere. For the oriented matroid of the complete graph K_n, we show that the face poset of the "coarse" subdivision of $\mathcal{B}^+(K_n)$ is dual to the face poset of the associahedron A_{n-2}. This is joint work with Federico Ardila (MSRI) and Lauren Williams (MIT).

10:30 AM **Coffee break**

11:00 AM **Adriano M. Garsia (University of California, San Diego)**

On the m-quasi-invariants of S_3

We introduce here a new approach to the study of m-quasi-invariants. This approach consists in representing m-quasi-invariants as N-tuples of invariants. Then conditions are sought which characterize such N-tuples. We study here the case of S_3 m-quasi-invariants. This leads to an interesting free module of triplets of polynomials in the elementary symmetric functions e_1, e_2, e_3 which explains certain observed properties of S_3 m-quasi-invariants. We also use basic results on finitely generated graded algebras to derive some general facts about regular sequences of S_n m-quasi-invariants. This is joint work with Nolan Wallach (University of California, San DIego) and Jason Bell (University of Michigan).

11:30 AM **Peter McNamara (LaCIM, UQAM)**

Cylindric Schur Functions

To describe our first motivation for cylindric Schur functions, we begin with a conjecture from Stanley's Ph.D. thesis on the symmetry of a generating function for P-partitions. The second motivation concerns the fundamental open problem of finding a combinatorial proof of the non-negativity of the 3-point Gromov-Witten invariants. After introducing these topics, we will discuss cylindric Schur functions from the point of view of Schur-positivity.

NOON **Lunch break**

2:00 PM **Gabor Hetyei (UNC Charlotte)**

Orthogonal polynomials represented by CW-spheres

Given a sequence $\{Q_n(x)\}_{n=0}^{\infty}$ of symmetric orthogonal polynomials, defined by a recurrence formula $Q_n(x) = \nu_n \cdot x \cdot Q_{n-1}(x) - (\nu_n - 1) \cdot Q_{n-2}(x)$ with integer ν_i's satisfying $\nu_i \geq 2$, we construct a sequence of nested Eulerian posets whose *ce*-index is a non-commutative generalization of these polynomials. Using spherical shellings and direct calculations of the *cd*-coefficients of the associated Eulerian posets we obtain two new proofs for a bound on the true interval of orthogonality of $\{Q_n(x)\}_{n=0}^{\infty}$. Either argument can replace the use of the theory of chain sequences. Our *cd*-index calculations allow to represent the orthogonal polynomials as an explicit positive combination of terms of the form $x^{n-2r}(x^2-1)^r$. Both proofs may be extended to the case the ν_i's are not integers and the second proof is still valid when only $\nu_i > 1$ s required. The construction provides a new "limited testing ground" for Stanley's non-negativity conjecture for Gorenstein* posets, and suggests the existence of strong links between the theory of orthogonal polynomials and flag-enumeration in Eulerian posets.

2:30 PM **Louis J. Billera (Cornell University)**

CW-spheres from meet-distributive lattices

Meet-distributive lattices are to convex closure what geometric lattices are to linear or affine span. In the latter case, when the underlying matroid is orientable, there is an associated Eulerian poset of faces of the corresponding arrangement of (pseudo) hyperplanes, whose enumerative information can be obtained directly from the geometric lattice. We show that this continues to hold for abstract convex closures: for each there is a regular CW-sphere whose enumerative information can be read in the same way from the underlying meet-distributive lattice of closed sets. In both cases, the transfer of information from lattice to sphere is accomplished by changing descents to peaks in an edge-labeling of the lattice. This is joint work with Samuel K. Hsiao (University of Michigan) and J. Scott Provan (University of North Carolina).

Friday, June 25

8:00 AM **Registration and continental breakfast**

9:00 AM **Bo-Yin Yang (Tamkang University, Tamsui, Taiwan)**

A Study in XL

With cryptanalyses announced for several stream ciphers and PKC's, "algebraic attacks" as a cracking tool are very much a hot topic, especially after the disputed Courtois-Pieprzyk "cryptanalysis of AES" proclamation. Controversy persists to this day.

In existing literature, simulations of these algorithms far outweighs theory. We believe that a comprehensive and systemic study of such "algebraic attacks", algorithms to solve a determined or overdetermined system of equations over a finite field and especially the XL and related algorithms initially introduced by Shamir *et al*, is long overdue.

From theoretical, empirical and practical viewpoints, we analyze the general applicability and performance of XL and variants. In the process, we paint a more accurate picture of how XL and variants work, derive rigorous conditions for their successful operation, and hence better security estimates, including a reasonable comparison between XL and Groebner basis methods.

We carry out the same analysis on other combinations of dimensions and base fields, and reach the conclusion that XL and variants represent a theoretical advance and still deserve careful study, but definitely do not work as well as it is alleged to.

As an example, the authors of SFLASH^{v2} recently revised its specifications to produce SFLASH^{v3}, with much bigger dimensions and keys. We are given to understand that an attack from the XL family was the reason. We examine possible attacks with several XL variants against SFLASH^{v2}, and the upshot is that Patarin *et al* might have sold their own design short, as the purported cryptanalysis can be proven inoperative.

We conclude by suggesting one or two better XL variants, and directions for future research. This is joint work with Jiun-Ming Chen (Chinese Data Security Inc., Taipei, Taiwan).

9:30 AM **Emden R. Gansner (AT&T Labs - Research)**

How to Draw a Graph

Graph theory is one of the mainstays of discrete mathematics, with a bevy of significant structural and algorithmic results. Perhaps overlooked is the more pedestrian task of drawing a graph. As it happens, this area has drawn from linear algebra, linear programming, computational geometry, and physics as well as the more obvious graph theory and algorithms. In this talk, we consider some ways in which these peripheral areas can be used, especially to draw hierarchies and large, undirected graphs.

10:00 AM **Karen L. Collins (Wesleyan University)**

Graph Homomorphisms and Graph Cores

A graph homomorphism is a map of between the vertices of two graphs, which preserves the edges. A graph core is a graph whose only graph homomorphisms to itself are automorphisms. Cores are the natural representatives of the equivalence classes of graphs in the lattice of graphs under graph homomorphisms. This talk will present several methods by which cores can be adapted to produce new cores.

10:30 AM **Coffee break**

11:00 AM **Julian West**

Perfect matching graphs for Gale-Robinson sequences

A domino is a rectangle of dimensions 1 by 2. An "Aztec diamond" of order n is a diamond-shape array of squares with height and width $2n$. The number of different ways of covering an Aztec diamond of order n with dominos is $2^{n(n+1)/2}$. There are a number of attractive proofs of this formula, notably the "condensation" proof used by Eric Kuo to verify the recursion formula. We generalize this proof to the "Gale-Robinson" recurrences, which have the form $a(n)a(n-m) = a(n-i)a(n-j) + a(n-k)a(n-l)$, where $i+j = k+l = m$. (The Aztec diamond recurrence corresponds to the case of $i = j = k = l = 1$.) In the process, we construct graphs analogous to the Aztec diamonds and in which the number of perfect matchings are given by the appropriate term in the Gale-Robinson sequence. This is joint work with Mireille Bousquet-Melou and Jim Propp.

11:30 AM **Francesco Brenti (University of Rome II)**

Special matchings and Kazhdan-Lusztig polynomials

In their fundamental paper [Representations of Coxeter groups and Hecke algebras, Invent. Math., 53 (1979), 165-184] Kazhdan and Lusztig defined, for every Coxeter group W, a family of polynomials, indexed by pairs of elements of W, which have become known as the Kazhdan-Lusztig polynomials of W. These polynomials are intimately related to the Bruhat order of W and have proven to be of fundamental importance in several areas of mathematics including representation theory, the geometry and topology of Schubert varieties, immanants, and quantum groups.

Our purpose in this talk is to show that the combinatorial concept of a special matching (which can be defined for any partially ordered set) plays a fundamental role in the computation of these polynomials. Our main result also implies the recent one in [F. Du Cloux, Rigidity of Schubert closures and invariance of Kazhdan-Lusztig polynomials, Advances in Math., 180 (2003), 146–175] about the combinatorial invariance of Kazhdan-Lusztig polynomials. More precisely, while the result of Du Cloux is non-constructive and holds for Coxeter systems whose Dynkin diagram is either a tree or affine of type A, our result is constructive and holds for all Coxeter systems. This is joint work with Fabrizio Caselli (Univ. Lyon I) and Mario Marietti (Univ. of Rome II).

NOON **Lunch break**

2:00 PM **Neil L. White (University of Florida)**

Lagrangian Matroids

Lagrangian matroids are a special class of Coxeter matroids. They are much more closely related to ordinary matroids than are arbitrary Coxeter matroids. We survey their properties, including basis exchange, circuit elimination, and representations.

2:30 PM **Patricia Hersh (University of Michigan)**

Chains of modular elements

Well before the introduction of the notion of EL-labeling, Richard Stanley provided a labeling for all supersolvable lattices which allowed him to express the Möbius function and related data in terms of the number of permutations labeling maximal chains on an interval which have various descent sets. His labeling was an EL-labeling, so one of the eventual consequences was that the order complex of any supersolvable lattice of rank n, i.e. any graded lattice with a maximal chain $\hat{0} < m_1 < \cdots < m_{n-1} < \hat{1}$ of modular elements, is at least $(n-3)$-connected (since it is shellable). In joint work with John Shareshian, we recently showed that any finite lattice with a (not necessarily maximal) chain $\hat{0} < m_1 < \cdots < m_r < \hat{1}$ of modular elements is at least $(r-2)$-connected, in some sense extending Stanley's result. This is joint work with John Shareshian (Washington University in St. Louis).

3:00 PM **Anders Björner (KTH, Stockholm)**

Geometrically constructed homology bases for certain partition lattices

We use the theory of hyperplane arrangements to construct natural bases for the homology of partition lattices of types A, B and D. This extends and explains the "splitting basis" for the homology of the partition lattice given by Wachs in 1996, thus answering a question asked by R. Stanley.

More explicitly, the following general technique is presented and utilized. Let A be a central and essential hyperplane arrangement in $\mathbb{R}^d$. Let $R_1, \ldots, R_k$ be the bounded regions of a generic hyperplane section of A. We show that there are induced polytopal cycles ρ_{R_i} in the homology of the proper part of the intersection lattice such that $\{\rho_{R_i}\}_{i=1,\ldots,k}$ is a basis for its homology. This geometric method for constructing combinatorial homology bases is applied to the Coxeter arrangements of types A, B and D, and to some interpolating arrangements. This is joint work with Michelle Wachs (Univ. of Miami, Coral Gables, FL).

4:00 PM **Ping-pong tournament**

Saturday, June 26

8:00 AM **Registration and continental breakfast**

9:00 AM **Dominique Foata (Strasbourg)**

Word and permutation statistics

The paper that will be read at the celebration of Richard Stanley's 60-th birthday falls inside the scope of the research, initiated by Adin, Brenti, Gessel, Reiner, Roichman, Stanley, Steingrímsson, Stembridge and others... whose purpose is to unveil the basic properties of the statistics defined on the symmetric groups, Coxeter groups and also rearrangements of words with repeated letters. The present paper will be focused on the equidistribution of the classical statistics "major index" and "inversion number" when the inverse line of route and the upper and lower record sequences are kept fixed. Analytic expressions in the algebra of the q-series are also derived.

9:30 AM **Einar Steingrímsson (Chalmers, Göteborg, Sweden)**

Decreasing subsequences in permutations and Wilf equivalence for involutions

In a recent paper, Backelin, West and Xin describe a map ϕ^* that repeatedly replaces the lexicographically least occurrence of the pattern $k\cdots 21$ in a permutation σ by an occurrence of the pattern $(k-1)\cdots 21k$. The resulting permutation $\phi^*(\sigma)$ contains no decreasing subsequence of length k.
For example, if $k=3$,

$$5\,\underline{3}\,\underline{2}\,6\,\underline{1}\,4 \quad\mapsto\quad \underline{5}\,\underline{2}\,\underline{1}\,6\,3\,4 \quad\mapsto\quad 2\,1\,5\,6\,3\,4 \quad=\quad \phi^*(5\,3\,2\,6\,1\,4)$$

In the BWX paper, the definition of ϕ^* is actually extended to full rook placements on a Ferrers board (where the permutations correspond to square boards), and this is used to prove the following result (and more): For any n and any pattern τ, the number of permutations of length n that avoid $12\cdots k\tau$ equals the number that avoid $k(k-1)\cdots 21\tau$.
We prove that ϕ^* commutes with taking the inverse of a permutation and thus ϕ^* takes involutions to involutions, which shows that the above result restricts to involutions, as recently conjectured by Jaggard.
There are some intriguing open questions and problems here, such as: Find a (simple) non-recursive description of the bijection ϕ^*, and: Why does repeated application of ϕ take an involution to an involution, although some non-involutions may be encountered on the way (as in the above example)? This is joint work with Mireille Bousquet-Mélou.

10:00 AM **Miklos Bona (University of Florida)**

Why one pattern is easier to avoid than another one

First, it was believed that all patterns of length k are avoided by roughly the same number of permutations of length n, in the sense that $L(q) = \lim(S_n(q))^{1/n} = (k-1)^2$ for all such patterns. While being Richard Stanley's student, I disproved this conjecture by providing a counterexample. However, nobody really understood why that counterexample worked, or could provide another counterexample. Now, eight years later, we will provide an infinite family of counterexamples, and start explaining why they work.

10:30 AM **Coffee break**

11:00 AM **Alexander Postnikov (MIT)**
Permutohedra, associahedra, and beyond
We give 3 different formulas for the volume of the permutohedron. The volume is given by a polynomial in n variables. The coefficients of this polynomial, which we call the mixed Eulerian numbers, are certain positive integer numbers that have many interesting combinatorial properties. Various specializations of these numbers produce such well-known integer sequences as the Eulerian numbers, the binomial coefficients, factorials, the numbers $(n+1)^{n-1}$ of trees, the Catalan numbers, and others. We give a combinatorial formula for the mixed Eulerian numbers in terms of plane binary trees.
We also study a more general class of polytopes that includes permutohedra, Stasheff's associahedra and cyclohedra, Stanley-Pitman polytopes, graphical zonotopes, and various generalized associahedra that came up in DeConcini-Procesi wonderful arrangements. We describe combinatorial structure of these polytopes and give formulas for their volumes and Erhart polynomials.

11:30 AM **Sergey Fomin (University of Michigan)**
Cluster algebras of classical type
I will discuss combinatorial constructions, main properties, and concrete examples of cluster algebras of classical types ABCD. Results, obtained jointly with Andrei Zelevinsky, include Laurentness, positivity, canonical bases, and description of the defining ideals.

NOON **Lunch break**

2:00 PM **Christos Athanasiadis (University of Crete)**
Ehrhart polynomials, simplicial polytopes and magic squares
We describe a certain class of integer polytopes P having the property that the polynomial $h(t)$ which appears as the numerator in the Ehrhart series of P, when written as a rational function of t, is equal to the h-polynomial of a simplicial polytope, so that the coefficients of $h(t)$ satisfy the conditions of the g-theorem. This class includes the order polytopes of graded posets, previously studied by Reiner and Welker, and the polytope of doubly stochastic $n \times n$ matrices. In the latter case the unimodality of the coefficients of $h(t)$, which follows, was conjectured by Stanley in 1983.

2:30 PM **Lynne Butler (Haverford College)**
On the area of cyclic polygons
Heron's formula, from 60 BC, for the area A of a triangle in terms of its side lengths a, b and c, states that $A^2 = s(s-a)(s-b)(s-c)$, where s is the semiperimeter $(a+b+c)/2$. In 1994 David Robbins published a similar formula for cyclic pentagons, 5-gons whose vertices are points on a circle. When he died in September 2003, he and Julie Roskies were working on the case of cyclic heptagons. Miller Maley stepped in and by March 2004 they established a formula for the area of a cyclic heptagon in terms of its side lengths. I report on this work of Maley, Robbins and Roskies. This is joint work with Miller Maley (Center for Communications Research), David Robbins (Center for Communications Research) and Julie Roskies (Center for Communications Research).

3:00 PM **Coffee break**

4:00 PM **Daniel J. Kleitman (Mathematics Department, MIT)**

Partitioning a Planar Graph into a Forest and Something Else

Given an arbitrary planar graph G, we show that it is possible to find two edges incident to each vertex such that removal of all these edges from G leaves a forest. Another way to say this is: Let H be the bipartite graph whose vertex sets are the vertices and edges of G with an edge between incident vertex-edge pairs of G. H is thus the graph whose adjacency matrix is the incidence matrix of G. Then for any planar graph G, H can be partitioned into a forest and a graph of maximum degree 2. The corresponding statement is not true for all graphs G that can be imbedded on a torus. The graphs H defined here are planar graphs of girth 6. An open question is: can any planar graph of girth 6 be so partitioned?

4:30 PM **Richard Stanley (Massachusetts Institute of Technology)**

Problems I couldn't solve

5:00 PM **Adjourn**