QUAL PREP SESSION 3

Problem 1

Let X be the vector space of polynomials in one (or more, you pick) variable with real (or complex, you pick) coefficients. It is easy to see that this is a vector space (real or complex depending on the coefficients). Prove that no matter what norm you define on X, it is never a Banach space.

Hint: Consider the subspace of polynomials of degree n or less.

Problem 2

Prove the *compact-Hausdorff* lemma by elementary methods (nothing but definitions is needed): Let $f: X \to Y$ be a continuous, injective map. Suppose X is compact and Y is Hausdorff. Then f is a homeomorphism (i.e. f^{-1} is continuous).

Problem 3

Let X be a normed linear space and S a subspace such that S is an open set in X. Prove that $X = S$.

Problem 4

Something to think about: Let X be an infinite dimensional Banach space. If by Alaoglu, the norm unit ball of X^* is weak-* compact, how come X^* is not LCH (locally compact Hausdorff) in the weak-* topology.

And yes it is Hausdorff, so that's not a problem. Further, there is a norm ball around every point of X^* that is weak-* compact by Alaoglu, so that's not a problem either.