QUAL PREP SESSION 5

Problem 1

Let T be defined by $\varphi \mapsto \varphi'(0)$ for $\varphi \in \mathcal{D}(\mathbb{R})$.

Show that T is a distribution and that T is not a (complex) measure.

Problem 2

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function that is C^1 on $\mathbb{R} \setminus \{0\}$ and further that the one sided limits of f exist at 0 (f has a jump discontinuity). Compute the distribution derivative of f.

Problem 3

True or false (prove or find a counterexample)

Suppose $f : \mathbb{R} \to \mathbb{R}$ is a strictly increasing continuous function (hence L^1_{loc} and further the derivative f' exists almost everywhere). Further assume that $f' \in L^1_{loc}$. Then (this implication is what is true or false) the distribution derivative of f is f' (that is the distribution induced by f').

Problem 4

Let $E \subset [0, 1]$. Suppose E is of the first category.

Is it true (prove or find counterexample) that necessarily $m(E) < 1$?