

QUAL PREP SESSION 7

Problem 1

Suppose $\mathbb{C}[x]$ (polynomials) is taken as a subspace of $C([0, 1])$ with the uniform norm. Find an example of an unbounded linear functional $T: \mathbb{C}[x] \rightarrow \mathbb{C}$

Problem 2

Is $C([0, 1])$ complete with respect to the $L^{1/2}$ metric? i.e. the metric (not a norm of course):

$$\rho(f, g) = \int_0^1 \sqrt{|f(x) - g(x)|} dx$$

Problem 3

Show that $L^1([0, 1])$ is separable.

Problem 4

Suppose $f_n, g \in L^2(\mathbb{R})$ and suppose that $\|f_n\| = \|g\| = 1$. Further suppose that $|1 - \int f_n g| \leq 1/n$. Show that f_n 's are not an orthonormal set (can't be a basis).

Problem 5

Show that linear combinations of functions of the form $\sin(ax)$ and $\cos(bx)$ ($a, b \in \mathbb{N}$) are dense in $L^2([-\pi, \pi])$
