Math 20E Requirement Fulfillment Exam

Practice Exam

Note: This is a practice exam. The actual exam may differ in terms of difficulty and the topics covered.

Instructions: Please read the instructions carefully.

- You have only one chance to take the Math 20E Requirement Fulfillment Exam! Do not take this exam if you are not prepared!
- Write your name above.
- Please write your answers in this booklet.
- You must show your work to get full credit.
- No calculators or notes are allowed.
- You have 40 minutes to complete the exam.
- Each problem is worth 10 points.

1. Consider the vector field

$$\mathbf{F} = 2xy\,\mathbf{i} + (x^2 + z)\,\mathbf{j} + y\,\mathbf{k}.$$

(a) Find a function $f = f(x, y, z)$ such that $\mathbf{F} = \mathbf{grad} f$.

(b) Find $\nabla \times \mathbf{F}$ (i.e. the curl of $\mathbf{F}$.)

(c) *Using your answer to (a)*, calculate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{R},$$

where C is the curve given by $\mathbf{R} = 2t\,\mathbf{i} - 3\sin\left(\frac{\pi}{2}t\right)\,\mathbf{j} + t^2\,\mathbf{k}$, for $0 \leq t \leq 1$.

2. Let $\mathbf{F}$ be the vector field

$$\mathbf{F} = 3x\,\mathbf{i} + xz^3\,\mathbf{j} - e^y\,\mathbf{k}.$$

Let B be the solid unit ball $x^2 + y^2 + z^2 \leq 1$, and let S be the boundary of B. If $\mathbf{n}$ is the outward facing unit normal to S then *use the Divergence Theorem* to calculate the surface integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS.$$

Hint: you may use the fact that the volume of B is $\dfrac{4\pi}{3}$.

3. Let S be the portion of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 0$ and let $\mathbf{F}$ be the vector field

$$\mathbf{F} = (z - y)\mathbf{i} + (x + z)\mathbf{j} - (e^{xyz} \cos y)\mathbf{k}.$$

Using Stoke's Theorem, find the surface integral

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS,$$

where $\mathbf{n}$ is the unit normal to S (the one that points in the direction of positive z.)