Real Algebraic Geometry in Matrix Variables

Abstract:

Where do polynomial inequalities come from? We have polynomials $p$ and $q$ acting on tuples $x$ of numbers in $\mathbb{R}^n$. Suppose $p(x) > 0$ for all $x$ in $\mathbb{R}^n$ making $q(x) > 0$. Is there some algebraic relationship between $p, q$ equivalent to this? That is a lot to hope for, but an “algebraic certificate” equivalent to an inequality often exists and this is the substance of much of real algebraic geometry (RAG). It is a subject which bloomed in the last 50 years.

Now consider $n$ tuples $X := \{X_1, \ldots, X_n\}$ of symmetric matrices $X_j$ and polynomials $p$ and $q$ acting on such tuples, for example, $n = 2$ and

$$p(X) := X_1 X_2^3 + X_2^3 X_1 + X_1^5.$$ 

The polynomial yields a value $p(X)$ that is a symmetric matrix, and we can consider the same issues as in classical RAG. We have polynomials $p$ and $q$. Suppose $p(X)$ is positive definite for all $X$, making $q(X)$ a positive definite matrix. Recall a positive definite matrix is one whose eigenvalues are all $> 0$. A theory parallel to RAG for converting such inequalities to algebra formulas has emerged in the last ten years. The motivation came from problems in systems engineering but spread out from this in many directions.

The talk will give a taste of selections from this smorgasboard.

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