Abstract:

The algebraic K theory space $K(R)$ is defined as a topological group completion, which on $\pi_0$ is just the usual algebraic group completion of a monoid which yields $K_0(R)$. Amazingly, it turns out that this space not only has a multiplication on it which is associative and commutative up to homotopy, but it is an infinite loop space. This means that it represents a spectrum (the stable analogue of a space), and therefore a cohomology theory. We construct equivariant algebraic K-theory for G-rings. However, spectra with G-action (called naive G-spectra) are not robust enough for stable homotopy theory, and the objects of study in equivariant stable homotopy theory are genuine G-spectra, which correspond to cohomology theories graded on representations.

Our construction of “genuine” equivariant algebraic K-theory recovers as its fixed points the K-theory of twisted group rings, and as particular cases equivariant topological real and complex K-theory, Atiyah’s Real K-theory and statements previously formulated in terms of naive G-spectra for Galois extensions. For example, we can reinterpret the map from the Quillen-Lichtenbaum conjecture and the assembly map from Carlsson’s conjecture in terms of genuine G-spectra and their fixed points.

We will not assume background in topology and will explain all the concepts from homotopy theory that arise in the talk.