A $p$-adically entire function with integral values on $\mathbb{Q}_p$ and the exponential of perfectoid fields

Abstract:

We give an essentially self-contained proof of the fact that a certain $p$-adic power series

$$\Psi = \Psi_p(T) \in T + T^2 \mathbb{Z}[[T]] ,$$

which trivializes the addition law of the formal group of Witt $p$-covectors $\widehat{CW}_p$, is $p$-adically entire and assumes values in $\mathbb{Z}_p$ all over $\mathbb{Q}_p$. We also carefully examine its valuation and Newton polygons. We will recall and use the isomorphism between the Witt and hyperexponential groups over $\mathbb{Z}_p$, and the properties of $\Psi_p$, to show that, for any perfectoid field extension $(K, ||)$ of $(\mathbb{Q}_p, ||_p)$, and to a choice of a pseudo-uniformizer $\varpi = (\varpi(i))_{i\geq 0}$ of $K^\times$, we can associate a continuous additive character $\Psi_\varpi : \mathbb{Q}_p \to 1 + K^{\infty}$, and we will give a formula to calculate it. The character $\Psi_\varpi$ extends the map $x \mapsto \exp \pi x$, where

$$\pi := \sum_{i \geq 0} \varpi(i) p^i + \sum_{i < 0} (\varpi(0)) p^{-i} \in K .$$

I will also present numerical computation of the first coefficients of $\Psi_p$, for small $p$, due to M. Candilera.

Host: Kiran Kedlaya

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