Abstract:
In 1844, Kummer showed that cyclotomic integer rings can fail to be principal ideal domains. In 1976 and 1984, Ribet and Mazur-Wiles used Galois representations attached to modular forms to partially describe class groups that measure the extent of this failure. The exact structure of these class groups remains a mystery to this day. I will explain how to attach ideal classes to geodesics in the complex upper half-plane. A conjecture of mine states these two constructions are inverse to each other in an appropriate sense. I hope to motivate a broader philosophy, developed jointly with Takako Fukaya and Kazuya Kato, that certain arithmetic objects attached to Galois representations of global fields can be described using higher-dimensional modular symbols.