Abstract:

Halmos once described problems and their solutions as the heart of mathematics. Following this line of thinking, one might naturally ask: What, then, is the heart of problems? In this talk, I attempt to answer this question using techniques from statistics, information visualization, and machine learning. I begin the journey by cataloging the features of problems delineated by the mathematics and mathematics education communities. These dimensions are explored in a large data set of students working thousands of problems at the Art of Problem Solving, an online company that provides adaptive mathematical training for students around the world. To increase the number of features of mathematical problems that can be studied, this quantitative exploration is partnered with a qualitative analysis that involves human scoring of 105 problems and their solutions. Using correlation matrices, principal components analysis, and clustering techniques, I explore the relationships among those features frequently discussed in mathematics problems (e.g., difficulty, creativity, novelty, affective engagement, authenticity). Along the way, I define a new set of uncorrelated features in problems and use these as the basis for a New Mathematical Problem Typology (NMPT). Grounded in the terminology of classical music, the NMPT works to quickly convey the essence and value of a problem, just as terms like étude and mazurka do for musicians.