Oka Principles and the Linearization Problem.

Abstract:
This is a talk for a general audience. Let $G$ be a complex Lie group and let $Q$ be a Stein manifold (closed complex submanifold of some $\mathbb{C}^n$). Suppose that $X$ and $Y$ are holomorphic principal $G$-bundles over $Q$ which admit an isomorphism $\Phi$ as topological principal $G$-bundles. Then the famous Oka principle of Grauert says that there is a homotopy $\Phi_t$ of topological isomorphisms of the principal $G$-bundles $X$ and $Y$ with $\Phi_0 = \Phi$ and $\Phi_1$ biholomorphic. We prove generalizations of Grauert’s Oka principle in the following situation: $G$ is reductive, $X$ and $Y$ are Stein $G$-manifolds whose (categorical) quotients are biholomorphic to the same Stein space $Q$.

We give an application to the Holomorphic Linearization Problem. Let $G$ act holomorphically on $\mathbb{C}^n$. When is there a biholomorphic map $\Phi : \mathbb{C}^n \to \mathbb{C}^n$ such that $\Phi^{-1} \circ g \circ \Phi \in \text{GL}(n, \mathbb{C})$ for all $g \in G$? We describe a condition which is necessary and sufficient for “most” $G$-actions.

This is joint work with F. Kutzschebauch and F. Larusson.