Abstract:

Gross showed that to every Hermitian symmetric tube domain we may associate a canonical variation of Hodge structure (VHS) of Calabi-Yau type. The construction is representation theoretic, not geometric, in nature, and it is an open question to realize this abstract VHS as the variation induced by a family of polarized, algebraic Calabi-Yau manifolds. In order for a geometric VHS to realize Gross’s VHS it is necessary that the invariants associated to the two VHS coincide. For example, the Hodge numbers must agree. The later are discrete/integer invariants. Characteristic forms are differential-geometric invariants associated to VHS (introduced by Sheng and Zuo). Remarkably, agreement of the characteristic forms is both necessary and sufficient for a geometric VHS to realize one of Gross’s VHS. That is, the characteristic forms characterize Gross’s Calabi-Yau VHS. I will explain this result, and discuss how characteristic forms have been used to study candidate geometric realizations of Gross’s VHS.