Math 209 - Number Theory

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One-parameter families of elliptic curves with non-zero average root number. Joint work with S. Bettin and C. Delaunay.

Abstract:

We investigate in this talk the average root number (i.e. sign of the functional equation) of non-isotrivial one-parameter families of elliptic curves (i.e. elliptic curves over $\mathbb{Q}(t)$, or elliptic surfaces over $\mathbb{Q}$). For most one-parameter families of elliptic curves, the average root number is predicted to be 0. Heilgott showed that under Chowla’s conjecture and the square-free conjecture, the average root number is 0 unless the curve has no place of multiplicative reduction over $\mathbb{Q}(t)$. We then build non-isotrivial families of elliptic curves with no place of multiplicative reduction, and compute the average root number of the families. Some families have periodic root number, giving a rational average, and some other families have an average root number which is expressed as an infinite Euler product. We then prove several density results for the average root number of non-isotrivial families of elliptic curves, over $\mathbb{Z}$ and over $\mathbb{Q}$ (the previous density results found in the literature were for isotrivial families). We also exhibit some surprising examples, for example, non-isotrivial families of elliptic curves with rank $r$ over $\mathbb{Q}(t)$ and average root number $-(-1)^r$, which were not found in previous literature.

Special Note:
There will be a pre-talk from 1:15-1:45pm.

Host: Alina Bucur

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