Abstract:

Let $*$ be a binary operation on a set $X$ and let $x_0, x_1, \ldots, x_n$ be $X$-valued indeterminate. Define two parenthesizations of $x_0 * x_1 * \cdots * x_n$ to be equivalent if they give the same function from $X^{n+1}$ to $X$. Under this equivalence relation, we study the number $C_{*,n}$ of equivalence classes and the largest size $e_{C_{*,n}}$ of an equivalence class. We have $1 \leq C_{*,n} \leq C_n$ and $1 \leq e_{C_{*,n}} \leq C_n$, where $C_n := \frac{1}{n+1} \binom{2n}{n}$ is the ubiquitous Catalan number. Moreover, $C_{*,n} = 1 \iff *$ is associative $\iff e_{C_{*,n}} = C_n$. Thus $C_{*,n}$ and $e_{C_{*,n}}$ measure how far the operation $*$ is away from being associative. In this talk we will present various results on the nonassociativity measurements $C_{*,n}$ and $e_{C_{*,n}}$, and show their connections to many known combinatorial results, assuming $*$ satisfies some multiparameter generalizations of associativity.