Abstract:

One way to measure the entanglement of a (pure) quantum state $\Psi$ is the Geometric Measure of Entanglement $E(\Psi)$, related to the spectral norm of tensor product states. The maximal possible value of $E(\Psi)$ on an $m$-qubit state $\Psi$ is $m$; it is 0 only for product states.

In quantum computation, it is tempting to think “the more entanglement”, the better. In 2009, Gross, Flammia, and Eisert showed that this intuition is incorrect. They proved that if $\Psi$ is an $m$-qubit state with near maximal entanglement, $E(\Psi) > m - \delta$, and if an NP problem can be solved by a computer with the power to perform local measurements on $\Psi$, then there is a purely classical algorithm that can solve the same problem (with positive probability) in a time only about $2^\delta$ times longer.

This suggests states with low entanglement are needed to get the exponential speed-up quantum computation is supposed to offer. However, as Gross et. al. also show, the situation seems hopeless: with respect to the Haar probability measure on all $m$ qubit states, $E(\Psi)$ is bigger than $m$ minus log factors with very high probability as $m$ grows.

Fortunately, this analysis ignores one key fact: the real quantum states that any proposed quantum computers use are Boson (symmetric states), since they are built out of photons. Hence, the results on entanglement of generic states do not apply.

In this talk, I will discuss my recent work with Shmuel Friedland, where we prove that the maximal possible entanglement for an $m$-qubit Boson state is $\log_2(m + 1)$. Moreover, we show the same concentration phenomenon in this sphere: up to $\text{double}$ log factors, with very high probability Boson quantum states are maximally entangled.