The classical congruence subgroup problem asks whether every finite quotient of $G = \text{GL}_n(\mathbb{Z})$ comes from a finite quotient of $\mathbb{Z}$. I.e. whether every finite index subgroup of $G$ contains a principal congruence subgroup of the form $G(m) = \ker(G \to \text{GL}_n(\mathbb{Z}/m\mathbb{Z}))$ for some $m \in \mathbb{N}$? If the answer is affirmative we say that $G$ has the congruence subgroup property (CSP). It was already known in the 19th century that $\text{GL}_2(\mathbb{Z})$ has many finite quotients which do not come from congruence considerations. Quite surprising, it was proved in the sixties that for $n \geq 3$, $\text{GL}_n(\mathbb{Z})$ does have the CSP.

Observing that $\text{GL}_n(\mathbb{Z}) \cong \text{Aut}(\mathbb{Z}^n)$, one can generalize the congruence subgroup problem as follows: Let $\Gamma$ be a group. Does every finite index subgroup of $G = \text{Aut}(\Gamma)$ contain a principal congruence subgroup of the form $G(M) = \ker(G \to \text{Aut}(\Gamma/M))$ for some finite index characteristic subgroup $M \leq \Gamma$? Very few results are known when $\Gamma$ is not abelian. For example, we do not know if $\text{Aut}(F_n)$ for $n \geq 3$ has the CSP. But, in 2001 Asada proved, using tools from algebraic geometry, that $\text{Aut}(F_2)$ does have the CSP, and later, Bux-Ershov-Rapinchuk gave a group theoretic version of Asada’s proof (2011).

On the talk, we will give an elegant proof to the above theorem, using basic methods of profinite groups and free groups.