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Math 243 - Functional Analysis Seminar

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Matrix Convex Sets Without Absolute Extreme Points

Abstract:

Let $M_n(\mathbb{S})^g$ denote g-tuples of $n \times n$ complex self-adjoint matrices. Given tuples $X = (X_1, \ldots, X_g) \in M_{n_1}(\mathbb{S}^g)$ and $Y = (Y_1, \ldots, Y_g) \in M_{n_2}(\mathbb{S})^g$, a matrix convex combination of X and Y is a sum of the form

$$V_1^* X V_1 + V_2^* Y V_2 \qquad V_1^* V_1 + V_2^* V_2 = I_n$$

where $V_1 : M_n(\mathbb{R}) \to M_{n_1}$ and $V_2 : M_n(\mathbb{R}) \to M_{n_2}$ are contractions. Matrix convex sets are sets which are closed under matrix convex combinations.

While in the classical setting there is only one good notion of an extreme point, there are three natural notions of extreme points for matrix convex sets: Euclidean, matrix, and absolute extreme points. A central goal in the theory of matrix convex sets is to determine if one of these notions of extreme points for matrix convex sets is minimal with respect to spanning.

Matrix extreme points are the most restricted type of extreme point known to span matrix convex sets; however, they are not necessarily the smallest set which does so. Absolute extreme points, a more restricted type of extreme points that are closely related to Arveson's boundary, enjoy a strong notion of minimality should they span. However, until recently it has been unknown if general matrix convex sets are spanned by their absolute extreme points.

This talk will give a class of closed bounded matrix convex sets which do not have absolute extreme points. The sets considered are noncommutative sets, K_X, formed by taking matrix convex combinations of a single tuple X. In the case that X is a tuple of compact operators with no nontrivial finite dimensional reducing subspaces, K_X is a closed bounded matrix convex set with no absolute extreme points.

Host: Adrian Ioana

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