Minimizers of the sharp Log entropy on manifolds with non-negative Ricci curvature and flatness

Abstract:
Consider the scaling invariant, sharp log entropy (functional) introduced by Weissler on noncompact manifolds with nonnegative Ricci curvature. It can also be regarded as a sharpened version of Perelman’s W entropy in the stationary case. We prove that it has a minimizer if and only if the manifold is isometric to the Euclidean space. Using this result, it is proven that a class of noncompact manifolds with nonnegative Ricci curvature is isometric to $\mathbb{R}^{n}$. Comparing with earlier well known flatness results on asymptotically flat manifolds and asymptotically locally Euclidean (ALE) manifolds, their decay or integral condition on the curvature tensor is replaced by the condition that the metric converges to the Euclidean one in $C^1$ sense at infinity. No second order condition on the metric is needed.