Math 288 - Probability & Statistics

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The maximum of the characteristic polynomial for a random permutation matrix

Abstract:
Let $P$ be a uniform random permutation matrix of size $N$ and let $\chi_N(z) = \det(zI - P)$ denote its characteristic polynomial. We prove a law of large numbers for the maximum modulus of $\chi_N$ on the unit circle, specifically,

$$\sup_{|z|=1} |\chi_N(z)| = N^{x_c + o(1)}$$

with probability tending to one as $N \to \infty$, for a numerical constant $x_c \approx 0.677$. The main idea of the proof is to uncover an approximate branching structure in the distribution of (the logarithm of) $\chi_N$, viewed as a random field on the circle, and to adapt a well-known second moment argument for the maximum of the branching random walk. Unlike the well-studied CUE field in which $P_N$ is replaced with a Haar unitary, the distribution of $\chi_N(z)$ is sensitive to Diophantine properties of the argument of $z$. To deal with this we borrow tools from the Hardy–Littlewood circle method in analytic number theory. Based on joint work with Ofer Zeitouni.

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