Department of Mathematics,
University of California San Diego

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MATH 288 - Probability & Statistics

Brian Hall

University of Notre Dame

Eigenvalues for Brownian motion in the general linear group

Abstract:

I will discuss four families of random matrices. The first two are classical: a Gaussian measure on the space of $N \times N$ Hermitian matrices ("Gaussian unitary ensemble") and a Gaussian measure on the space of all $N \times N$ complex matrices ("Ginibre ensemble"). As $N \to \infty$, the eigenvalues of the Gaussian unitary ensemble concentrate onto an interval with a semicircular density, while the eigenvalues of the Ginibre ensemble become uniformly distributed in a disk in the complex plane.

Now, the space of $N \times N$ Hermitian matrices can be identified with the Lie algebra $u(N)$ of the unitary group $U(N)$, and the Gaussian unitary ensemble is the distribution of Brownian motion in $u(N)$. Similarly, the space of all $N \times N$ matrices is the Lie algebra $gl(N; \mathbb{C})$ of the general linear group $GL(N; \mathbb{C})$ and the Ginibre ensemble is the distribution of Brownian motion in $gl(N; \mathbb{C})$. It is then natural to consider also Brownian motions in the groups $U(N)$ and $GL(N; \mathbb{C})$ themselves.

The eigenvalues for Brownian motion in $U(N)$ have a known limiting distribution in the unit circle. The eigenvalues for Brownian motion in $GL(N; \mathbb{C})$ have received little attention up to now. Assuming that the eigenvalues have a limiting distribution, recent results of mine with Kemp show that the limiting distribution is supported in a certain domain Σ_t in the complex plane. The figure shows the domain for $t = 3.85$, along with a plot of the eigenvalues for $N = 2,000$. One notably feature of the domains is that they change topology from simply connected to doubly connected at $t = 4$. I will give background on all four families of random matrices, describe our new results, and mention some ideas in the proof.

Host: Todd Kemp

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