Nondegeneracy in CR Geometry

Abstract:
CR geometry studies boundaries of domains in $\mathbb{C}^n$ and their generalizations. In characterizing CR structures, a central role is played by the Levi form $L$ of a CR manifold $M$, which measures the failure of the CR bundle to be integrable, so that when $L$ has a nontrivial kernel of constant rank, $M$ is foliated by complex manifolds. If the local transverse structure to this foliation still determines a CR manifold $N$, $M$ is called straightenable, and the Tanaka-Chern-Moser classification of CR hypersurfaces with nondegenerate Levi form can be applied to $N$. It remains to classify those $M$ for which $L$ is degenerate and no such straightening exists. This was accomplished in dimension 5 by Ebenfelt, Isaev-Zaitzev, Medori-Spiro, and Pocchiola. I will discuss their results, my progress on the problem in dimension 7, and my work (joint with Igor Zelenko) modifying Tanaka’s prolongation procedure to treat the equivalence problem in arbitrary dimension.